

Measuring vulnerability to multidimensional poverty

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A method for measuring vulnerability to multidimensional poverty is proposed under a mean-risk behavior approach. We extend the unidimensional downside mean-semideviation measurement of vulnerability to poverty towards the multidimensional space by incorporating this approach into the multidimensional Alkire and Foster counting framework. The main indicator generated by this method is called the VMPI alluding to the fact that it can be used for the assessment of vulnerability to poverty measured by the Multidimensional Poverty Index (MPI). The proposed family of vulnerability indicators allows to be estimated using cross-sectional data and can include both binary and metric indicators. It is flexible enough to be applied for measuring vulnerability in a wide range of multidimensional poverty indexes design, including the Global MPI. An empirical application of the VMPI and its related indicators is illustrated using cross-sectional data from the households Socioeconomic Characterization Survey of Chile 2017.

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I. Introduction

The multidimensional nature of welfare has been strongly posited in research literature. This trend has spread to various areas of welfare economics and has a strong presence in both the distributional analysis (e.g., Kolm, 1977; Atkinson and Bourguignon, 1982; Maasoumi, 1986; Tsui, 1995; Decancq and Lugo, 2012) and static analysis of poverty (e.g., Anand and Sen, 1997; Mukherjee, 2001; Tsui, 2002; Atkinson, 2003; Bourguignon and Chakravarty, 2003; Chakravarty and Silber, 2008; Asselin, 2009; Alkire and Foster, 2011; Alkire and Santos, 2014; Belhadj and Limam, 2012; Bossert et al., 2013). In recent years, this multidimensional perspective has been also extended to the study of poverty dynamics (e.g., Nicholas and Ray, 2012; Alkire et al., 2017; Alkire, Roche and Vaz, 2017) and pro-poor growth (e.g., Klasen, 2008; Grosse et al., 2008; Berenger and Bresson, 2012).

Concerning the analysis of vulnerability to poverty, the literature remains

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behind in following this multidimensional view of human well-being. In the majority of recently published research on vulnerability, the unidimensional perspective continues to prevail (see for instance, Dutta et al., 2011; Calvo and Dercon, 2013; Gallardo, 2013; Klasen and Waibel, 2013; Celedoni 2013 and 2015; Günther and Maier, 2014; Povel, 2015; Hohberg et al., 2018, among others). However, some attempts to expand knowledge on vulnerability toward the multidimensional view have been made. Calvo (2008) extended the Calvo and Dercon (2005, 2007) utility approach of vulnerability to the multidimensional space. Feeny and McDonald (2015), following the approach of vulnerability as expected poverty (see e.g: Chaudhuri et al., 2002; Christiaensen and Subbarao, 2005), proposed a vulnerability to multidimensional poverty index as the probability of being multidimensionally poor. In contrast, Abraham and Kavi (2008), instead of extending a one-dimensional vulnerability approach to the multidimensional space, proposed a fuzzy measure of multidimensional vulnerability. It consists of applying a fuzzy poverty approach to a set of several welfare dimensions that are then aggregated with equal weights. As a result, a fuzzy index of multidimensional poverty is obtained, which is associated with multidimensional vulnerability.

This paper contributes to the research on poverty by proposing a new family of vulnerability to multidimensional poverty indexes (VMPI) under a mean-risk behaviour approach. We extend the unidimensional Gallardo (2013) measurement of vulnerability to poverty towards the multidimensional space through the incorporation of this approach in the multidimensional ‘AF counting method’ (Alkire and Foster, 2011). In our design, the downside mean-semideviation is used as the risk parameter for each welfare dimension.

The proposed method allows the application using cross-sectional data, which constitutes an important practical advantage in comparison with other multidimensional vulnerability measures that are only applicable with panel data (e.g.: Calvo, 2008). The proposed multidimensional measurement can also include both binary and metric indicators, and it is flexible enough to be applied for measuring vulnerability in a wide range of multidimensional poverty AF frameworks, including the global Multidimensional Poverty Index (MPI) regularly estimated by the United Nations Development Program with the technical support of the Oxford Poverty and Human Development Initiative (OPHI).

The proposed VMPI shares the advantages and disadvantages of the AF family of multidimensional poverty indicators. The parsimonious nature of the measurement allows it to be accessible to practical public policy purposes, which is its main advantage. However, like any parsimonious model, there is a cost to be assumed in abstracting some complexities of reality. In particular, the AF method is a simplified model at the cost of abstracting from the existing interdependencies between the welfare dimensions. That is, this family of indicators follows a procedure of counting and averaging the deprivations, treating them as if they were independent of each other. In reality, these dimensions are not entirely independent of each other. We know for instance, that for a person whose main productive resource is his labour force, being employed is a condition to obtain an income and a satisfactory standard of living, and this in turn depends on enjoying

good health. It is obvious that a state of malnutrition affects children's health and their ability to learn and be successful in schooling. We could mention several similar situations in which it seems a truism that some dimensions of well-being generate effects on other dimensions. Nevertheless, there are at least three solid arguments that support the use of the AF method as a suitable tool for analysing multidimensional poverty.

The first argument comes from Sen's capability approach, according to which, functionings are states of 'being and doing', which people value intrinsically (see for instance Sen, 2003). Therefore, the welfare dimensions should not be seen in a simplistic manner, as if they were goods or resources that can be traded, substituted or complemented in a utilitarian way. Enjoying good health, being educated, having a job or living in a suitable home in a safe environment with your family are some of the aspects of human life that people value intrinsically independently from each other.

A second argument comes from the empirical evidence: when poverty has been measured in multiple welfare dimensions, there is insufficient evidence of a strong association between such dimensions. Conversely, such an association is usually weak.

The third argument is probably the strongest as it comes from practice. The real value of a measurement is not being capable of capturing all the complexity of reality, but instead being sufficiently useful and understandable to be applied in practice by policymakers. A measure of poverty is significant when it is widely used to make real decisions and have positive impact on poor people's lives. The AF method may have some shortcomings because of its simplification, but it has been very successful in being widely used in practice, and it is having a real impact on public policies, thus generating positive changes in the lives of people suffering poverty around the world. The global MPI currently covers 105 countries, which represents 77% of the world's population. Other countries have developed their own national MPIs with specific designs using the same AF method. Therefore, a VMPI that could be computed with cross-sectional data and could be applied within the AF framework, could have a real impact on public policies.

This paper makes a contribution to the scarce research literature on vulnerability to multidimensional poverty. The development of such a measurement is crucial for public policy purposes, because it is applicable to understanding the multidimensional nature of existing poverty but also the multidimensional nature of the risk of becoming or remaining poor. Vulnerability is not only about the uncertainty existing with a low monetary income; it also concerns deprivations in non-monetary dimensions of welfare (e.g., health, education, employment, housing and public security). Policymakers must design strategies to reduce the risk of falling into poverty in multiple dimensions; therefore, they need to utilise an appropriate measurement of multidimensional vulnerability.

The ideal data set to measure vulnerability to poverty is, of course, a long, balanced panel. However, data sets of this nature do not usually exist in the real world in which the public policies of developing countries operate. In these countries, it is common practice to measure poverty using cross-sectional data. Panel data are scarce in the developing world, and, even when they are available,

they are usually short in the time dimension and unbalanced due to the typical panel attrition problem. Thus, given such practical limitations and with the hope that the proposed measurement could have a real impact on public policies of developing countries, we have decided to offer an empirical illustration as realistic as possible using a recent cross-sectional data set for a medium income level developing country. The method is empirically illustrated using cross-sectional data from the National Socioeconomic Characterization Survey (CASEN) from Chile 2017.

In what follows, we briefly review the concept of unidimensional vulnerability to poverty according to the main approaches in the literature, then, the proposed measurement framework is presented, after that, we offer the empirical illustration and conclude with some final remarks.

II. Unidimensional vulnerability: a brief review

When we observe the stochastic process of a household's consumption expenditure, we say that a household is poor in one period of time whenever the effective realization of such a stochastic process is below the so called 'poverty line' in that period. In turn, the 'poverty line' is a monetary threshold that means having enough to acquire a consumption basket that satisfies the food needs and other basic household current expenses. In that sense, poverty is an observable and easily verified fact. The only thing we need to identify a person as poor is simply to record whether it meets the condition $y_{it} < z$, where y_{it} is his consumption expenditure in the time period t , and z is the monetary threshold that defines the poverty line.

Vulnerability to monetary poverty instead concerns the uncertainty people face of having consumption expenditure below the poverty line during a period of time. That is, unlike poverty, vulnerability is not an effective realization of the consumption stochastic process, but rather concerns the risk of being poor.

Note, however, that such a risk, although it constitutes a real threat of obtaining a low well-being level, is not necessarily realized in a state of effective poverty. In short, if the monetary poor people are those whose effective consumption expenditure is below the poverty line, the vulnerable people are those who are at risk of having a consumption expenditure below that threshold. Thus, since vulnerability concerns only a probable event, not an effective one, it is not easily to measure and verify. As a result, many measures of vulnerability have emerged in the literature on poverty, and no one measure has achieved predominance over the others. The complexity of identifying vulnerable people among a population, is dealt with in a recent literature review article (Gallardo, 2018) in which ten of these vulnerability approaches are analysed and twelve different criteria are found to identify those at risk of poverty.

The approaches of vulnerability to monetary poverty can be classified today into four groups (see also Gallardo, 2018; Hoddinott and Quisumbing, 2003): Vulnerability as uninsured exposure to risk (VER), Vulnerability as expected poverty (VEP), Vulnerability as low expected utility (VEU) and Vulnerability by mean-risk (VMR).

A. Vulnerability as uninsured exposure to risk (VER)

For several researchers, vulnerability is essentially a situation of uninsured exposure to risk. This is also the view of the World Bank (2001, p. 139), according to which, ‘vulnerability measures the resilience against a shock - the likelihood that a shock will result in a decline in welfare’. In this concept, a household must be considered ‘vulnerable’, whenever it is unable to avoid a consumption fall in the face of adverse income shocks. Among the scholars who have contributed to this line of research on vulnerability, there are different approaches. Some of them (e.g., Glewwe and Hall, 1999; Dercon and Krishnan, 2000; Amin et al., 2003; Skoufias and Quisumbing, 2005) have defined vulnerability in terms of the household’s inability to maintain a consumption stream over time to cope with income variability. Others associate vulnerability with an extending poverty line in a sufficient monetary amount to cover an insurance instrument that could be used for achieving the poverty line as certainty equivalent (Cafiero and Vakis, 2006). There are also two approaches in this group that define vulnerability in terms of uninsured exposure to downside risk (Povel, 2015; Klasen and Waibel, 2013; Dutta et al., 2011). That is, as uninsured exposure to experience well-being loss from a reference level, which could be the current consumption (Povel, 2010; Klasen and Waibel, 2013) or even a hybrid threshold between the current consumption and the poverty line (Dutta et al., 2011).

The concept of vulnerability as an extended poverty line (Cafiero and Vakis, 2006) also belongs to this group of approaches. According to this concept, to identify those people as vulnerable to poverty, we must proceed in a similar way as we do when identifying the poor, but with a slightly higher poverty line, which acts as vulnerability threshold. This threshold must be defined so that its gap with respect to the current poverty line is equal to the cost of insurance that eliminates the risk of deviating from an expected consumption over the poverty line. Cafiero and Vakis (2006) formally expressed such a vulnerability threshold as $z + \eta_i$, where z is the conventional poverty line and η_i is the cost of insurance to protect the household against the ‘socially unacceptable risk’ of falling below a current welfare level equal to z .

B. Vulnerability as expected poverty (VEP)

The expected poverty approach (e.g: Pritchett et al., 2000; Chaudhuri et al., 2002; Suryahadi and Sumarto, 2003; Christiaensen and Subbarao, 2005) is probably the most widely applied framework in the unidimensional vulnerability assessment, both by scholars and policymakers and practitioners. According to this approach, a household must be considered vulnerable in the time period t when its probability of being poor in the future is above the probability threshold p^v . This probability threshold p^v defines the relevant risk to be classified in vulnerability conditions. It has been typically put at 0.5 (Pritchett et al., 2000, p. 5; Suryahadi and Sumarto, 2003, p. 48), because ‘if a household is just at the poverty line and faces a mean zero shock, then this household has a one period ahead vulnerability of 0.5’. But in addition, it is ‘intuitive to say a household is “vulnerable” if it faces at least 50% probability of falling into poverty’, because

at this point it has the same chances of being poor and not being poor. A higher probability of being poor implies that the chances of being poor are greater than the chances not being poor. Recently, Hohberg et al. (2018) aimed to determine the optimal probability threshold for this probability approach using a Receiver Operating Characteristic Analysis (ROC), which allows one identify the optimal probability cut-off for an outcome predictor of a binary variable, which in this case is being poor.

C. Vulnerability as low expected utility (VEU)

The low expected utility approach associates vulnerability to the expected utility gap with respect to utility on the poverty line. Different variations of this approach have been proposed in the literature. The simplest is the Ligon and Schechter (2003) formulation: $v_i = U(z) - E[U(y_i)]$, where v_i is the vulnerability of household i , $U(z)$ is the utility on poverty line under certainty, and $E[U(y_i)]$ is the expected utility of household consumption $U(y_i)$. That is, under this approach, vulnerability is the gap of expected utility to the poverty line. Notice also that in this perspective, the relevant risk to be poor is measured through Jensen's Inequality according to the curvature of the utility function $U(y_i)$. This can be seen easily if we express the same formulation as follows:

$$(1) \quad v_i = [U(z) - U(E(y_i))] - [U(E(y_i)) - E(U(y_i))]$$

Where in the first bracket we have the expected poverty utility gap and in the second one we have Jensen's Inequality as the risk component of vulnerability.

Calvo and Dercon (2013) proposed a variation of this approach in which they introduced the element of downside risk by defining the relevant outcome focusing on the consumption expenditure censored in the poverty line. The expected utility is censored in the poverty line as follows: $E[U(\text{Min}\{y_i, z\})]$. They also specified two utility functions that allow one to fulfil a desirable axiomatic basis for measuring vulnerability to poverty.

Günther and Maier (2014) contributed with a more general variation of this approach, which is supported on the reference-dependent utility theory (Kahneman and Tversky, 1979). Instead of measuring the vulnerability gap with a conventional utility function, they proposed performing the vulnerability assessment using a 'reference utility function', which also includes a gain-loss component with respect to an initial reference point. The idea behind this proposal is that the utility loss of an individual in the face of a welfare fall always depends on the reference utility point in which this person was before experiencing the loss.

D. Vulnerability by mean-risk (VMR)

In the literature, two unidimensional vulnerability approaches concur in being structured under mean-risk models. They are the Chiwaula-Witt-Waibel (2011) approach that is based on the expected value of the focal variable and its standard

deviation, and the Gallardo (2013) approach which is supported in a mean-risk dominance criterion with the use of the typical downside mean-semideviation as the risk parameter. In the Chiwaula-Witt-Waibel (2011) approach, vulnerable people are those whose expected outcome, discounting a standard deviation σ_{y_i} , is below the poverty line. That is, the vulnerable people fulfil the following condition: $E(y_i) - \sigma_{y_i} < z$. While in the Gallardo (2013) approach, the vulnerable people are those whose expected consumption, discounting a weighted standard downside mean-semideviation $\sigma_{y_i}^-$, is below or equal to the poverty line: $E(y_i) - \gamma \sigma_{y_i}^- \leq z$, where the weight $\gamma \in [0, 1]$ is a risk aversion parameter.

III. The proposed measurement

The standard strategy for measuring poverty in a population was defined by Sen (1976, p. 219) as following a biphasic pattern consisting of: ‘(i) identifying the poor among the total population and (ii) constructing an index of poverty using the available information on the poor.’ The first component is known in the literature as ‘the identification problem,’ and the second is known as the ‘aggregation problem’. The same strategy must be followed to measure vulnerability to poverty.

In the unidimensional approach to poverty, the identification problem is usually solved in one cut-off through a poverty line z . A household is classified as poor whenever its focal welfare variable (consumption or income) is less than z . In the multidimensional approach, by contrast, two cut-offs are required (see Alkire and Foster, 2011; Bourguignon and Chakravarty, 2003; Bossert et al., 2013): one to determine who is welfare-deprived in each dimension (unidimensional identification) and another to determine who must be considered poor in the multidimensional space (multidimensional identification). A similar ‘two-cut-off’ strategy is proposed here to address the identification problem for the VMPI.

The first cut-off is performed through a generalization of the procedure proposed by Gallardo (2013) for unidimensional vulnerability, which is explained in the following scenario. Consider a population of N individuals with sub-indices $i = 1, 2, \dots, N$. Suppose we have defined M of welfare’s dimensions, which are denoted by the sub-indices $m = 1, 2, \dots, M$. The welfare of each person over a period of time t can be represented by the vector $y_i = (y_{i1}, \dots, y_{iM})$, where each $y_{im} \in \mathbb{R}_+$ is a random variable of the welfare outcome for person i in the focal attribute m . The array of all the vectors in the population, forms the random matrix Y of N rows and M columns.

The random variables in vector y_i could be metrical or categorical-binary. All binary variables in a y_i vector follow a Bernoulli probabilistic process. In the event of deprivation, the binary variable y_{im} is equal to zero; in the presence of the welfare attribute, it is equal to one. If there are metrical variables in vector y_i , their probability density functions are unknown to the researcher. Thus, in the presence of metrical variables, the joint distribution $f(y_{i1}, \dots, y_{iM})$ is also unknown. However, we are always in a position to reasonably estimate $\mu_i = (\mu_{i1}, \dots, \mu_{iM})$ and $r_i = (r_{i1}, \dots, r_{iM})$, which are the vectors of the expected values of y_i and the risk parameter vector respectively, where each element $r_{im} \in \mathbb{R}_+$ such that $r_{im} \leq \mu_{im}$ represents the risk for y_{im} to deviate below the expected

value μ_{im} .

We assume that the individual's preferences can be reasonably represented by the utility function: $U_i(\mu_{i1}, \dots, \mu_{iM}; r_{i1}, \dots, r_{iM})$, with partial derivatives $\frac{\partial U_i}{\partial \mu_{im}} > 0$, $\frac{\partial U_i}{\partial r_{im}} < 0, \forall i, \forall m$. Therefore, for any two random vectors y_i, y_j , the following mean-risk dominance relation holds ($\succ$ denotes strict preference), with at least one inequality strict for some m :

$$(2) \quad y_i \succ y_j \Leftrightarrow \mu_{im} \geq \mu_{jm} \wedge r_{im} \leq r_{jm}, \forall m$$

We further assume the policymaker defines a vector of social preferences over the mean-risk relationships through the following vector of risk aversion coefficients: $\gamma = (\gamma_1, \dots, \gamma_M), \gamma_m \in (0, 1] \forall m$. There are two reasons why it is reasonable for each γ_m to be bounded in the interval $(0, 1]$. The first one, pointed out in Gallardo (2013), concerns the preference rationality. Due to the risk aversion preferences, we have $\gamma_m > 0, \forall m$. However, it is also reasonable to expect that the policymaker and society will value the gains in expected welfare at least as much as avoiding the risk losses. Therefore, $\gamma_m \leq 1, \forall m$. The second argument from Ogryczak and Ruszczyński (1999, 2001) is of a statistical nature. They demonstrated that, for the unidimensional relation $y_{im} \succ y_{jm} \Leftrightarrow (\mu_{im} - \gamma_m r_{im} > \mu_{jm} - \gamma_m r_{jm})$, the γ_m values bounded in the considered interval between zero and one, lead to a rational preference ordering, which is consistent with the second-order stochastic dominance criterion. Then, the assumption regarding the vector γ , allows us to compare any two random vectors y_i and y_j according to the following mean-risk dominance criterion:

$$(3) \quad y_i \succ y_j \Leftrightarrow (\mu_{im} - \gamma_m r_{im} \geq \mu_{jm} - \gamma_m r_{jm}) \forall m \wedge \exists (\mu_{im} - \gamma_m r_{im} > \mu_{jm} - \gamma_m r_{jm})$$

In the context of this preference framework, we extend the Gallardo (2013) definition of unidimensional vulnerability to establish the following unidimensional identification criteria.

Criterion 1 (unidimensional identification): *Let z_m be the vulnerability threshold for the welfare focal attribute y_m . Then, person i is vulnerable to poverty in dimension m whenever $\mu_{im} - \gamma_m r_{im} \leq z_m^v$. If y_m is a metrical variable z_m^v is equal to z_m , the poverty line in the focal attribute m under certainty. If y_m is a binary variable, z_m^v is equal to a probability threshold.*

The variable z_m^v is equal to a probability threshold for the binary variables because the expected value for such a type of variable is a probability. For these variables, the poverty threshold is equal to one under certainty and takes the value of zero when a household is poor in that focal dimension of welfare. However, for any Bernoulli variable, the expected value falls in the range $(0, 1)$ and is equal to the probability of this variable being equal to one. Thus, the value $\mu_{im} - \gamma_m r_{im}$

is also defined in the interval $(0, 1)$ for binary variables due to $\gamma_m \in (0, 1]$ and $r_{im} \leq \mu_{im}$.

Because the relevant risk is asymmetric in nature, we follow Gallardo (2013) in taking the standard downside mean-semideviation as the risk parameter in each dimension m . This parameter, which is widely used in financial research literature on risk assessment (see e.g., Estrada, 2002), is defined as follows:

$$(4) \quad \sigma_{im}^- = \sqrt{E \left\{ \min [(y_{im} - \mu_{im}), 0]^2 \right\}}$$

Then, for each Bernoulli variable, the standard downside mean-semideviation takes the following specific form:¹

$$(5) \quad \sigma_{im}^- = \sqrt{p_{im}^2 (1 - p_{im})}$$

where p_{im} is the probability of being not poor for person i in the dimension m , which in this case is also equal to the expected value of y_{im} . That is, for the Bernoulli welfare variables, the criterion 1 adopts the following specific form: *an individual i is vulnerable to poverty in the dimension m whenever* $p_{im} - \gamma_m \sqrt{p_{im}^2 (1 - p_{im})} \leq z_m^v$.

From this point, we move now to the AF method (Alkire and Foster, 2011; Alkire et al., 2015, chapter 5) to solve the problems of multidimensional identification and the aggregation in the summary multidimensional vulnerability measures.

Let $z^v = (z_1^v, \dots, z_M^v)$ be the vector of vulnerability thresholds for the relevant M focal welfare attributes. Let g_{im}^{v0} be an indicator function that is worth one when person i is vulnerable in the dimension m and let $w = (w_1, \dots, w_M)$ be the weighting vector for the M welfare dimensions, such that $\sum_{m=1}^M w_m = 1$. Then, the vulnerability score for household i is defined as the weighted sum:

$$(6) \quad s_i^V = \sum_{m=1}^M w_m g_{im}^{v0}$$

Using this vulnerability score, the individuals vulnerable to multidimensional poverty can be identified according to the following criterion:

¹The formula in (4) follows from the calculation of the standard semi-variance for these type of variables, where the event of being non-poor in the dimension m takes the value of one with a probability of p_{im} . Then the semi-variance of the Bernoulli variable y_{im} is obtained as follows: $E \left\{ \min [(y_{im} - \mu_{im}), 0]^2 \right\} = 0^2 p_m + (0 - p_m)^2 (1 - p_m) = p_m^2 (1 - p_m)$.

Criterion 2 (multidimensional identification): *The person i is vulnerable to poverty in the multidimensional space $\mathbb{R}^M$ whenever $s_i^V \geq k$, where k is the multidimensional poverty threshold.*

Regarding the choice of the multidimensional cut-off k , we recall that in the literature on multidimensional poverty there are two extreme solutions: the ‘union method’ and the ‘intersection method’. The former consists of classifying as multidimensionally poor those who are deprived in at least one welfare dimension. This method has the disadvantage of classifying too many people as poor. The majority of people in developing countries will be classified as poor using this rule because this extreme solution is lax in adopting the multidimensional threshold. For its part, the ‘intersection method’ consists of classifying as multidimensional poor those people who are deprived in all dimensions. This method, on the contrary, has the disadvantage of classifying very few people as multidimensionally poor because it adopts overly strict identification criterion. Thus, following the AF method, the best alternative is to choose an intermediate cut-off point. However, to define such a cut-off we must carry out an analysis of several alternatives that could be chosen in a reasonable range. It is possible to argue that such a choice will imply certain arbitrariness, but even the choice of the monetary poverty line, so commonly used by public policy makers, is not free of arbitrariness. The decision to find a reasonable range for k is a technical matter that falls under the specialists’ sphere. The final decision of choosing which k will be used to measure the multidimensional poverty in a country is ultimately the policymakers’ responsibility.

Once the vulnerable to multidimensional poverty individuals are identified, the next step is to solve the aggregation problem of quantifying in a summary VMPI the amount of multidimensional vulnerability existing in a population. The simplest multidimensional vulnerability measure will be the headcount ratio, i.e., the percentage of people vulnerable to multidimensional poverty in a population. This measure can be defined as follows:

$$(7) \quad V^H = \frac{1}{N} \sum_{i=1}^N I_{s_i^V \geq k}$$

where $I_{s_i^V \geq k}$ is an indicator function that equals one if person i is vulnerable to multidimensional poverty and otherwise equals zero. This measure, however, has several limitations such as the non-compliance of several desirable axioms and its inability to capture the intensity of vulnerability. To accomplish the task of defining a more general and suitable family of aggregate multidimensional vulnerability measures, we follow a more general procedure suggested by Alkire et al. (2015, p. 173-175) for summarizing cardinal deprivation indicators under the AF method.

We can define the normalized vulnerability gap of order α , for person i , in the dimension m as follows:

$$(8) \quad g_{im}^\alpha = g_{im}^0 \left(\frac{z_m^v - (\mu_{im} - \gamma_m r_{im})}{z_m^v} \right)^\alpha, \alpha \geq 0$$

Then, we define the following general vulnerability to multidimensional poverty measure of α order, belonging to the AF family of multidimensional measures and therefore associated with the FGT class (Foster, Greer and Thorbecke, 1984) of poverty measurements:

$$(9) \quad V^\alpha(Y, \gamma, z^v, k) = \frac{1}{N} \sum_{i=1}^N \sum_{m=1}^M w_m g_{im}^\alpha I_{s_i^v \geq k}, \alpha \geq 0$$

For $\alpha = 0$, the VMPI as adjusted headcount ratio (adjusted by the percentage of vulnerability deprivations) is obtained. For $\alpha = 1$ and $\alpha = 2$, the adjusted VMPI gap V^1 and the adjusted VMPI quadratic gap V^2 are respectively obtained. We recall that according to Alkire et al. (2015), all the AF family measures as defined in (9) satisfy the following desirable axiomatic properties for a poverty measurement: unidimensional deprivation focus, multidimensional deprivation focus, symmetry, replication invariance, scale invariance, dimensional monotonicity, population subgroup decomposability, dimensional breakdown and weak deprivation rearrangement (see the proofs in Alkire and Foster, 2011).

IV. Empirical application

A. Data

To illustrate the proposed measurement method, we offer an estimate of VMPI for Chile using cross-sectional data collected by the National Socioeconomic Characterization Survey (CASEN, 2017). This survey has been periodically conducted by the Chilean Ministry of Social Development since 1990, initially in triennial form and then on a biannual basis since 2009. This household survey has national coverage and is representative of all regions and the majority of municipalities in Chile. The purpose of the survey is to gather relevant information for assessing the socio-economic situation of Chilean households. The CASEN household's survey has a complex sample design stratified by conglomerates, also called segments (sections and blocks), which correspond to the primary sampling units (PSU). These conglomerates were formed by adhering to different grouping criteria, both in terms of limits and size. They correspond to groups of dwellings located in geographical areas defined by limits of streets, passages or agglomerations of private dwellings formed from one or more populated entities.

In 2017, the CASEN survey managed to interview 70,948 households with a total of 216,439 people. After filtering the data set for the missing data in some indicators, for this study a final sample of 70,837 households and 214,095 people

was obtained. In addition to the household's CASEN Survey, census data were also used in the econometric estimations for some variables. The latter data were taken from the Population Census of Chile of 2017. Table 1 shows the descriptive statistics of the data used in the vulnerability estimates.

TABLE 1—DESCRIPTIVE STATISTICS OF DATA USED IN THE ESTIMATES.

Indicator	Observations	Mean	Standar error	Coefficient of variation %
Variables at individual level	Individuals			
People per room (no more than 2.5)	214,095	0.9510	0.0019	0.2023
Housing materials (satisfactory materials)	214,095	0.9061	0.0022	0.2392
Housing tenure (regular occupation)	214,095	0.9240	0.0018	0.1992
Hot water availability	214,095	0.8201	0.0031	0.3779
Public security	214,095	0.7133	0.0063	0.8830
Schooling (children's attendance and adult's achievement)	214,095	0.8378	0.0031	0.3708
Good health status	214,095	0.7653	0.0030	0.3903
Employment (at least the 50% of working age people are employed)	214,095	0.8742	0.0019	0.2119
Income above the poverty line	214,095	0.9140	0.0019	0.2047
Variables at household level	Households			
Household head years of schooling	70,837	10.79	0.0551	0.5106
Household head age	70,837	53.22	0.1756	0.3300
Household head woman	70,837	0.42	0.0029	0.6929
Household size	70,837	3.04	0.0117	0.3860
Household with rural location	70,837	0.13	0.0032	2.5348
Household head indigenous	70,837	0.08	0.0018	2.1696
Dependence rate	70,837	2.21	0.0240	1.0847
Variables at municipality level	Census population			
Percent of indigenous population	target population	12.8%		
Average years of schooling	target population	11.9		
Percent of people working in mining sector	target population	1.4%		
Percent of people working in manufactory sector	target population	6.4%		
Percent of people working in agricultural sector	target population	6.0%		

B. Multidimensional poverty indicator design

Multidimensional vulnerability refers to the risk of being multidimensionally poor, so a focal multidimensional poverty design is required. It demands a suitable selection of the welfare dimensions, indicators and weights. Previously, using the AF method, four multidimensional poverty indicators (MPI) were designed for Chile with different purposes. Battiston et al. (2013) proposed an MPI design for Chile, homologated among six Latin American countries (Argentina, Brazil, Chile, El Salvador, Mexico and Uruguay), with the purpose of studying the changes in multidimensional poverty for the period 1992-2006. They proposed an MPI design with indicators in six equally weighted dimensions of well-being: income, school attendance for children, education of the household head, sanitation, water and shelter.

Santos and Villatoro (2018), meanwhile, proposed another regional MPI design for seventeen Latin American countries, including Chile. They included five welfare dimensions (housing, basic services, living standards, education and employment and social protection) and thirteen indicators (housing materials, people per room, housing tenure, adequate water source, sanitation, access to electricity, income, durable goods, children's school attendance, schooling gap, schooling achievement, employment and social protection). This second MPI

indicator was created to allow a regional comparison for several Latin American countries and to perform comparisons over time.

A third MPI design for Chile was proposed by Denis, Gallegos and Sanhueza (2010) with the purpose of being used at the national level in drafting public policies for poverty reduction. This third design included five dimensions (education, health, employment, housing and income) and nineteen indicators. A version of this last design is the national MPI used in official statistics. This indicator initially incorporated four dimensions with equal weights (education, health, work/social security, housing/environment), to which an additional dimension was added recently (networks and social cohesion) with a lower weight. This official design has a total of fifteen indicators: school attendance, school lag, schooling of adults, children's malnutrition, ascription to a health security system, health care, employment, ascription to a retirement system, retirement pension for older people, people per room, housing materials, basic services, support networks and social participation, security and equal treatment.

The first two MPI proposals are restricted to the data availability of other Latin American countries, meanwhile the last two MPI designs are not sufficiently parsimonious. Thus, for the purposes of this work, a new MPI design was developed for Chile by taking elements from the four previous designs, but adhering to the following normative and statistical criteria:

- 1) *Normative relevance*: the selected dimensions must be judged morally inescapable in order to be free to choose a decent life.
- 2) *National applicability*: our MPI design is country-specific, that is, it is not restricted to indicators that are comparable with other countries in the world, or with Latin American countries in particular, as is the case of those proposed by Batiston et al. (2013) and Santos and Villatoro (2018).
- 3) *Overall population coverage*: the selected indicators must be applicable to the entire population. A limitation of the MPI designs proposed by Denis et al. (2010) as well as the one used in the official statistics is that they use some indicators that are only applicable to a part of the population. The official MPI, for instance, uses an indicator of malnutrition that is only applicable to households with children (children's malnutrition); meanwhile, the pension for retirement indicator is only applicable to households with elderly adults. The problem with this type of indicator is that they exclude an important part of the population in the calculation of poverty in such dimensions. In the two examples just mentioned, households without children or without older people are excluded from the poverty assessment in these welfare dimensions. The consequence is a misleading calculation of poverty since it is not measured on the basis of the entire population. This also leads to an underestimation of multidimensional poverty. If children's and adult's malnutrition were measured together under a single concept of malnutrition, one could identify the malnourished households among a population. However, the CASEN survey does not have data availability on adult nutrition.

- 4) *Relevant incidence rate*: when an indicator has a very small incidence rate given the country development level, its inclusion in the national MPI is not relevant. For a medium income developing country such as Chile, it would not make sense to include, for instance, children's mortality or illiteracy as indicators because these phenomena are not frequently observed in Chile. In the official MPI for instance, some indicators have a very low incidence rate, such as ascription to a health system, since those not assigned to the private system have the option of subscribing to the public system called FONASA. Incorporating such an indicator into the multidimensional poverty measure leads to an underestimation of multidimensional poverty again.
- 5) *Susceptibility to policy intervention*: the selected indicators must be susceptible to being affected by policy intervention. An indicator that cannot be affected by policy intervention or that can be very little affected by it lacks real relevance in practice.
- 6) *Time stability*: the selected indicators should not be constructed from occasionally observable facts. For example, although the lack of medical care constitutes a deprivation, it cannot be registered as deprivation based on a temporary lack of a medical attendance for a cold or one off minor ailments.
- 7) *Parsimony*: the design of an MPI indicator must be simple and comprehensible to be easy to communicate and manage in public policy practices. From our viewpoint, the Chilean national MPI is too complex since it covers fifteen indicators, some of which are not relevant.
- 8) *Low association with other indicators*: the selected indicators must be weakly associated with each other. If the association between two indicators is high, then the independence assumption on which the MPI is constructed according to the AF method will not be tenable. Likewise, when two indicators have a very strong association, one of them would be redundant. As a measure of association between indicators, Cramer's V coefficient is used. For the case of two binary variables, this coefficient ranges in the interval $[-1, 1]$ and it is equivalent to the Mathews correlation coefficient also called 'phi coefficient'. A Cramer's V close to zero indicates that the association between two binary indicators is weak, whereas a high Cramer's V (e.g., with an absolute value of 0.5 or more) between two indicators would not be acceptable since it shows a strong association.

Based on the above indicated criteria, nine indicators to represent seven welfare dimensions were selected for this application: housing, living standards, education, health, public security, employment and income. Table 2 below shows the selected indicators, with their corresponding cut-off points and their uncensored and censored headcount ratios for each deprivation expanded to the population by using the complex sample design weighting of the CASEN survey. As it is common practice in MPI calculations around the world and in Chile, the selection of equal weights for each dimension was chosen. The alternative of choosing the weights based on factor analysis techniques has been questioned in

the literature due to its data-dependent nature (see Decancq and Lugo, 2013 for an informed discussion on this topic).

The Cramer's V coefficients for the comparisons of the binary variables of people deprived in each dimension are presented in Table 3. None of these coefficients exceeds 0.25 in absolute value, which provides evidence of a weak association among the selected indicators.

TABLE 2—STRUCTURE OF THE MULTIDIMENSIONAL POVERTY INDICATOR

Dimension	Indicator	Poverty cut-off	Weight	Uncensored headcount ratio	Censored headcount ratio
Housing	People per room	Overcrowding: there are more than 2.5 people per bedroom in the house	1/21	0.0490	0.0252
	Housing materials	Precarious housing: the material of the walls, roof or floor are precarious	1/21	0.0939	0.0464
	Housing tenure	Irregular housing occupation: the dwelling occupation is illegal or without a formal leasing agreement.	1/21	0.0760	0.0336
Living standard	Hot water availability	Household lacks a system to heat water	1/7	0.1799	0.1309
Public security	Public security	Shootings or fights have been high frequently observed in the neighborhood during the last month	1/7	0.2867	0.1595
Schooling	Schooling	At least one child between 6-17 do not attends school or at least the 50% of the adults do not completed the compulsory education	1/7	0.1622	0.1277
Health	Good health status	At least one person in the household suffers a chronic disease or declared to have a bad or very bad health status	1/7	0.2347	0.1461
Employment	Employment	50% or more of working age persons in the household are unemployed	1/7	0.1258	0.0948
Income	Income	The household percapita income is below the poverty line	1/7	0.0860	0.0740

TABLE 3—CRAMER'S V COEFFICIENTS FOR THE SELECTED MPI INDICATORS

Household deprived in...	People per room	Housing materials	Housing tenure	Hot water availability	Public security	Schooling	Health	Employment
Housing materials	0.0796							
Housing tenure	0.0846	0.0785						
Hot water availability	0.1115	0.1741	0.1065					
Public security	0.0462	0.0327	0.0334	-0.0171				
Schooling	0.0903	0.1265	0.0742	0.2471	0.0394			
Health	0.0337	0.0274	-0.0321	0.0235	0.0767	0.0423		
Employment	0.0160	0.0292	-0.0071	0.0735	0.0167	0.0998	0.0853	
Income	0.0892	0.0848	0.1594	0.2041	0.0093	0.1696	0.0212	0.2332

With respect to the multidimensional cut-off choice, in the baseline scenario, a threshold of 2/7 was selected and supported in the analysis of the results obtained for seven thresholds from 1/7 to 1, in 1/7 intervals. The 1/7 threshold yields to very high multidimensional poverty rates for a middle-income country like Chile. The thresholds greater than 2/7 yield even to lower multidimensional poverty headcount ratios than the official one. The 2/7 cut-off is equivalent to being deprived in at least two dimensions of poverty. In addition, this cut-off is in an intermediate position between the threshold of the Global MPI (33%) and that of Santos and Villatoro (2018) which was set at 25%, as well as the cut-off initially stated for the official national estimate (25% but recently adjusted to 22.5%).

However, both poverty and vulnerability measures were also estimated for the 1/7 and 3/7 multidimensional cut-off.

C. Vulnerability estimation strategy

In this illustrative example, all indicators are treated as Bernoulli type binary variables. In the case of income, the metric variable is dichotomized in the poor and non-poor categories, according to the official monetary poverty lines. These poverty lines are differentiated according to the household size to take into account the intra-household economies of scale. Treating the income variable as a binary allows us to estimate the income vulnerability with cross-sectional data following an analogous but reverse procedure to that proposed by Chaudhury et al., (2002). The procedure is analogous to that because it is also based on the probability estimation, but it is the reverse of it because, instead of estimating the unfavourable outcome (the probability of being poor), what is estimated here is the favourable one (the probability of being not-poor). This application is an innovation compared to the panel data estimation of monetary vulnerability implemented in the work of Gallardo (2013).

Our identification unit is the household. For each indicator variable we perform a multilevel Probit estimation with random intercepts in which the latent variable y_{im}^* is specified as follows:

$$(10) \quad y_{ijm}^* = \beta_0 + \beta_1 x_{ij} + \beta_2 z_j + u_j + e_{ij}$$

where x_{ij} is a vector of the household's characteristics including the household's head characteristics; z_j is a vector of municipality variables that affect the expected values in each welfare indicator; β_0 , β_1 and β_2 are the vectors of parameters; $u_j \sim N(0, \sigma_j^2)$ is a random intercept for each municipality j , while $e_{ij} \sim N(0, \sigma_{ij}^2)$ is a specific disturbance.

In addition to the probit model, we also performed logistic regression estimations under the same latent variable model specified in (10) but within the framework of logistic distributions. As we shall see later, both the probit and logit estimates give similar results, as is the case in other applications. However, the reason we also implement the logit estimates is because the logistics distribution has heavier tails than the normal distribution, which could be relevant in this context of vulnerability to poverty analysis. Therefore, we should verify whether the differences between these distributions in the tails will not significantly affect the results.

The probit and logit provide estimates for each probability p_{im} , which in turn are estimates of the expected value for each Bernoulli distributed indicator. Then we also use these probabilities for estimating the standard downside mean-semideviation following the formula defined above in (5). Once we have estimated both the mean and risk parameters, we are able to identify the vulnerable households for each indicator following the rule:

$$(11) \quad \hat{p}_{im} - \gamma_m \sqrt{\hat{p}_{im}^2 (1 - \hat{p}_{im})} \leq z_{im}^v$$

Where $\hat{p}_{im}$ is an estimate of p_{im} . However, we still need to reveal the procedure by which we define the vulnerability thresholds z_m^v and the risk aversion parameters γ_m . To solve the first, following Hohberg et al. (2018), we took advantage of the Receiver Operating Characteristic curve (ROC curve) analysis.

The ROC curve is a useful tool to find the threshold in which a binary predictor performs the best, i.e. to be as close as possible to the perfect prediction point. In the ROC space, we have in the ordinate axis the true positives rate (TPR) achieved for a binary predictor, against the false positive rate (FPR) in the abscissa axis.

In our framework, the TPR is the ratio between the truly predicted as non-poor people in each dimension divided by the total of effectively non-poor in the same dimension. The FPR is the ratio between the falsely predicted as non-poor among the total poor. The ROC curve is setting in the ROC space by plotting the achieved TPR and FPR points by a binary predictor for every prediction threshold. The perfect prediction point in the ROC space is found in the coordinates (0, 1) in Figure 1, in which the TPR equals one and the FPR equals zero. The optimal prediction threshold for a binary predictor is then at that point over the ROC curve which is closest to the perfect prediction point (Youden, 1950).

Figure 1 shows an illustration of the ROC curve for the income dimension. In keeping with the usual ROC analysis terminology, the TPR is called the ‘Sensitivity’ of the binary predictor. Meanwhile the FPR is called ‘1-Specificity’, given that the complement of the FPR is the true negative rate TNR (in our case the poor correctly predicted as poor), which is called the ‘Specificity’. The arrows in the figure indicate the point of perfect prediction in the ROC space and the point of best prediction over the ROC curve. This last point is what determines the choice of the optimal threshold to be predicted as non-poor in this dimension of well-being.

The argument that supports the choice of vulnerability thresholds using the ROC analysis is that using such a threshold maximizes the chances of a household being correctly predicted as non-poor in each dimension given their characteristics. Conversely, those households well predicted as non-poor will be unlikely to be predicted as poor in that dimension; thus, those households well predicted as non-poor are those that overcome the relevant risk of being poor in that dimension.

To choose the vulnerability threshold using the ROC curve analysis, we do not use the probabilities $\hat{p}_{im}$, but instead we directly use the discounting by downside risk predictor: $\hat{p}_{im} - \sqrt{\hat{p}_{im}^2 (1 - \hat{p}_{im})}$, with the risk aversion parameter γ_m fixed at one. Then, as we proceeded before in selecting the cut-off k , we calculated the multidimensional vulnerability indicator for different γ_m in a relevant range. We implement this analysis in the range of γ_m between 0.6 and 1. After that, we chose a unique risk aversion parameter equal to 0.8 as the most reasonable for

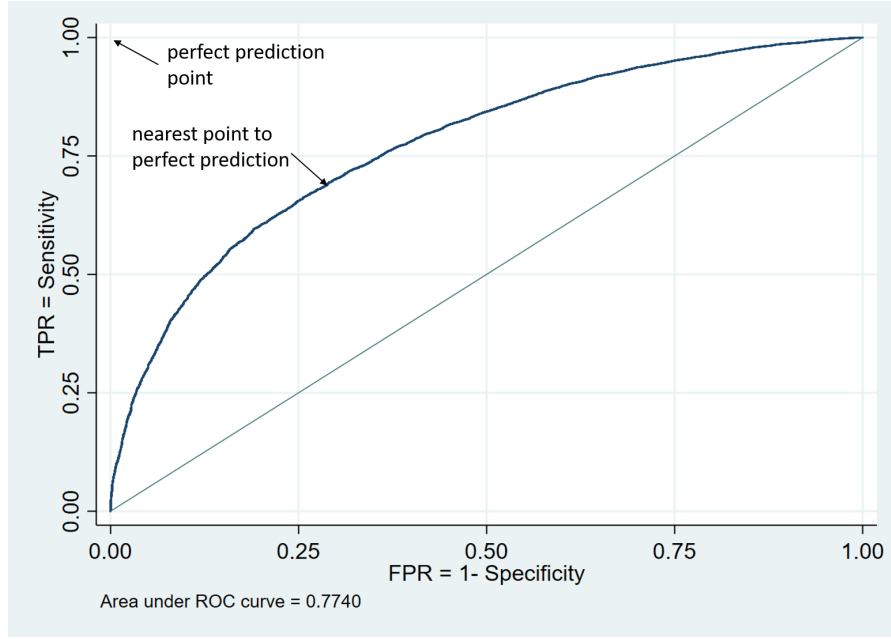


FIGURE 1. ROC CURVE FOR THE PROBIT PREDICTOR OF NON-POOR IN THE INCOME DIMENSION

all welfare dimensions. The option of choosing different risk aversion parameters in the different dimensions is also possible, although it is at the cost of losing parsimony and generating a more complex design.

D. Main results: MPI and VMPI estimates

Tables 4 and 5 present the econometric results for the probit and logit estimates. We found that variables of the household' head human capital (their schooling and age) have a positive effect on the probability of obtaining a well-being outcome above poverty in all dimensions. We also found that the probability of being non-poor in each well-being indicator is usually linked to other household's characteristics such as household size, rural location, belonging to an indigenous ethnic group, the gender of the household head and the dependency rate. All the random intercepts of the municipalities were found statistically significant in both the probit and logit estimates. While among the municipalities' control variables, the most important by its statistical significance is the average schooling of adults. Table 6 presents the vulnerability thresholds in each dimension obtained through the ROC curve analysis.

Table 7 shows the main results for multidimensional poverty and VMPI estimations for three different multidimensional cut-offs. According to the proposed MPI design and the selected multidimensional cut-off (2/7), 29.4% of Chileans are multidimensionally poor people and 43% are vulnerable to multidimensional poverty. It is also observed that the probit and logit estimates

TABLE 4—PROBIT ESTIMATION RESULTS

Variables	People per room	Housing materials	Housing tenure	Hot water availability	Public security	Schooling	Health	Employment	Income
Household's variables									
Household head years of schooling	0.0500*** (0.0034)	0.0583*** (0.0030)	0.0371*** (0.0039)	0.1070*** (0.0031)	0.0365*** (0.0029)	0.2294*** (0.0031)	0.0159*** (0.0014)	0.0243*** (0.0018)	0.0697*** (0.0020)
Household head age	0.0509*** (0.0043)	0.0087** (0.0027)	0.0328*** (0.0040)	0.0178*** (0.0026)	-0.0174*** (0.0023)	0.0192*** (0.0030)	-0.0229*** (0.0020)	-0.0353*** (0.0032)	0.0244*** (0.0033)
Household head age squared	-0.0003*** (0.0000)	0.0000 (0.0000)	-0.0000 (0.0000)	-0.0000 (0.0000)	0.0002*** (0.0000)	0.0001*** (0.0000)	0.0001*** (0.0000)	0.0002*** (0.0000)	-0.0000 (0.0000)
Household head woman	0.0031 (0.0199)	0.0049 (0.0147)	0.0481** (0.0158)	0.0182 (0.0181)	-0.0575*** (0.0119)	0.0561*** (0.0139)	-0.0834*** (0.0115)	-0.1617*** (0.0137)	-0.2348*** (0.0159)
Household size	-0.2451*** (0.0127)	0.0134* (0.0053)	0.0751*** (0.0078)	0.0200*** (0.0053)	-0.0412*** (0.0055)	-0.0166** (0.0056)	-0.11274*** (0.0042)	-0.0299*** (0.0051)	-0.0980*** (0.0067)
Household with rural location	-0.0142 (0.0388)	-0.2656*** (0.0537)	0.2918*** (0.0419)	-0.4958*** (0.0353)	0.8223*** (0.0359)	-0.2012*** (0.0178)	0.0737*** (0.0189)	-0.0702*** (0.0204)	-0.1975*** (0.0228)
Household head indigenous	-0.0801 (0.0422)	-0.0923* (0.0447)	0.0113 (0.0245)	-0.2033*** (0.0285)	-0.1004*** (0.0212)	-0.0640** (0.0246)	0.0007 (0.0189)	0.0086 (0.0259)	-0.0564* (0.0261)
Dependence rate	-0.0120* (0.0051)	-0.0060** (0.0023)	-0.0025 (0.0029)	-0.0065** (0.0022)	-0.0036 (0.0022)	0.0980*** (0.0039)	-0.0182*** (0.0021)	0.0668*** (0.0029)	-0.0157*** (0.0029)
Municipality's variables									
Percent of indigenous population	-0.2871* (0.1318)	0.5900* (0.2323)	-0.4526*** (0.1164)	-1.5516*** (0.1895)	0.4066* (0.1872)	0.0551 (0.0675)	0.0641 (0.0867)	0.1278 (0.0791)	0.0820 (0.1088)
Average years of schooling	-0.0329 (0.0232)	-0.0578 (0.0439)	0.0143 (0.0218)	0.1644*** (0.0345)	-0.0530 (0.0461)	0.0552*** (0.0132)	-0.0139 (0.0131)	0.1072*** (0.0134)	0.1820*** (0.0213)
Percent of people working in mining sector	-0.5641 (0.4471)	-2.1690*** (0.6582)	-1.1806* (0.5780)	1.1931** (0.4252)	-1.0196 (0.8437)	0.1673 (0.2259)	1.0275** (0.3166)	-0.4528 (0.2420)	0.9701** (0.3537)
Percent of people working in manufacturing sector	-0.1417 (0.8423)	1.0260 (0.9117)	-1.3810 (0.7122)	-0.6719 (1.3205)	-2.5200* (1.2519)	0.1175 (0.3689)	-0.5442 (0.4303)	0.9987* (0.4978)	1.5711* (0.6531)
Percent of people working in agricultural sector	-0.2002 (0.2431)	-1.1561** (0.3837)	0.1847 (0.2168)	0.7938* (0.3540)	1.5841*** (0.4743)	0.1135 (0.1305)	-0.2995* (0.1449)	0.6886*** (0.1491)	0.3954 (0.2105)
constant	0.9480** (0.3315)	0.9332 (0.5604)	-0.8161** (0.3088)	-2.7196*** (0.4766)	1.4224* (0.6083)	-3.2435*** (0.2200)	2.2350*** (0.1880)	0.8030*** (0.2037)	-2.2429*** (0.2807)
Municipality									
var(_constant)	0.0349*** (0.0090)	0.1179*** (0.0215)	0.0389*** (0.0060)	0.1827*** (0.0157)	0.2073*** (0.0215)	0.0042** (0.0014)	0.0176*** (0.0030)	0.0141*** (0.0026)	0.0492*** (0.0058)

Standard errors in parenthesis. *p value ≤ 0.1 , **p value ≤ 0.05 , ***p value ≤ 0.01 .

TABLE 5—LOGIT ESTIMATION RESULTS

Variables	People per room	Housing materials	Housing tenure	Hot water availability	Public security	Schooling	Health	Employment	Income
Household's variables									
Household head years of schooling	0.1094*** (0.0076)	0.1132*** (0.0065)	0.0764*** (0.0078)	0.1884*** (0.0049)	0.0660*** (0.0051)	0.4468*** (0.0056)	0.0271*** (0.0023)	0.0433*** (0.0035)	0.1378*** (0.0041)
Household head age	0.1067*** (0.0096)	0.0176*** (0.0051)	0.0434*** (0.0083)	0.0326*** (0.0045)	-0.0321*** (0.0041)	0.0399*** (0.0052)	-0.0432*** (0.0036)	-0.0696*** (0.0065)	0.0439*** (0.0065)
Household head age squared	-0.0005*** (0.0001)	0.0000 (0.0000)	0.0002* (0.0001)	-0.0000 (0.0000)	0.0004*** (0.0000)	0.0003*** (0.0000)	0.0002*** (0.0000)	0.0004*** (0.0001)	0.0000 (0.0001)
Household head woman	-0.0021 (0.0432)	0.0098 (0.0287)	0.0936*** (0.0318)	0.0294 (0.0323)	-0.0972*** (0.0203)	0.1257*** (0.0254)	-0.1397*** (0.0202)	-0.2841*** (0.0248)	-0.4672*** (0.0317)
Household size	-0.5220*** (0.0283)	0.0271* (0.0106)	0.1545*** (0.0158)	0.0346*** (0.0095)	-0.0702*** (0.0098)	-0.0121 (0.0102)	-0.2177*** (0.0074)	-0.0523*** (0.0094)	-0.1888*** (0.0123)
Household with rural location	-0.0210 (0.0874)	-0.5123*** (0.1029)	0.5995*** (0.0890)	-0.8415*** (0.0602)	1.6041*** (0.0723)	-0.3421*** (0.0310)	0.1240*** (0.0327)	-0.1359*** (0.0373)	-0.3739*** (0.0433)
Household head indigenous	-0.1697 (0.0897)	-0.1803* (0.0836)	0.0223 (0.0463)	-0.3406*** (0.0472)	-0.1785*** (0.0377)	-0.1024* (0.0436)	0.0013 (0.0335)	0.0164 (0.0486)	-0.1057* (0.0500)
Dependence rate	-0.0216 (0.0123)	-0.0109* (0.0044)	-0.0103 (0.0062)	-0.0100* (0.0039)	-0.0056 (0.0040)	0.1930*** (0.0070)	-0.0320*** (0.0036)	0.1308*** (0.0056)	-0.0315*** (0.0059)
Municipality's variables									
Percent of indigenous population	-0.5928* (0.2801)	1.2000** (0.4345)	-0.8913*** (0.2261)	-2.6446*** (0.3290)	0.8013* (0.3581)	0.1085 (0.1158)	0.1134 (0.1512)	0.2208 (0.1446)	0.2187 (0.2064)
Average years of schooling	-0.0568 (0.0523)	-0.1016 (0.0572)	0.0344 (0.0430)	0.3221*** (0.0602)	-0.1084 (0.0858)	0.1224*** (0.0208)	-0.0237 (0.0228)	0.2065*** (0.0251)	0.3785*** (0.0433)
Percent of people working in mining sector	-1.3454 (0.9551)	-4.1596*** (1.2376)	-2.1735* (1.1063)	1.9548** (1.7504)	-2.0873 (1.5371)	0.1085 (0.4154)	1.8787** (0.5863)	-0.8694* (0.4392)	1.7672* (0.6907)
Percent of people working in manufactory sector	-0.6649 (1.8517)	1.8418 (1.7180)	-2.6898 (1.3788)	-1.1692 (2.3052)	-4.8583* (2.3723)	0.1959 (0.6511)	-0.9176 (0.7496)	1.9180* (0.9216)	2.9701* (1.2555)
Percent of people working in agricultural sector	-0.4152 (0.5326)	-2.0896** (0.7290)	0.4063 (0.4224)	1.5637* (0.6179)	2.8280*** (0.8730)	0.3750 (0.2181)	-0.5203* (0.2507)	1.3081*** (0.2765)	0.9502* (0.4032)
constant	1.3763 (0.7506)	1.4064 (1.0886)	-1.5784** (0.6076)	-5.2542*** (0.8264)	2.6504* (1.1331)	-6.8771*** (0.3402)	3.9030*** (0.3279)	1.3810*** (0.3923)	-4.8671*** (0.5575)
Municipality var.(constant)	0.1615*** (0.0413)	0.4261*** (0.0726)	0.1433*** (0.0221)	0.5554*** (0.0491)	0.6928*** (0.0708)	0.0100* (0.0042)	0.0535*** (0.0089)	0.0500*** (0.0089)	0.1848*** (0.0217)

Standard errors in parenthesis. *p value ≤ 0.1 , **p value ≤ 0.05 , ***p value ≤ 0.01 .

TABLE 6—VULNERABILITY THRESHOLDS FOR EACH WELFARE INDICATOR

	People per room	Housing materials	Housing tenure	Hot water availability	Public security	Schooling	Health	Employment	Income
Using Probit estimates	0.817	0.611	0.671	0.388	0.509	0.487	0.466	0.638	0.611
Using Logit estimates	0.824	0.611	0.686	0.403	0.526	0.525	0.483	0.667	0.633

provide very similar results in the multidimensional vulnerability measures. This provides evidence in favour of being able to freely use either of the two econometric models to estimate VMPI. It also draws attention to the fact that the estimates of VMPI are quite precise judging by the standard error magnitudes.

TABLE 7—MAIN RESULTS AND COMPARISONS FOR DIFFERENT MULTIDIMENSIONAL CUT-OFF

	H	A	$M_0 - V^0$	V^1	V^2
Poverty					
k=1/7	0.6362 (0.0051)	0.2543 (0.0014)	0.1618 (0.0019)		
k=2/7	0.2938 (0.0045)	0.3734 (0.0015)	0.1097 (0.0019)		
k=3/7	0.1075 (0.0027)	0.5026 (0.0021)	0.0540 (0.0014)		
Vulnerability estimates with probit models					
k=1/7	0.7326 (0.0039)	0.3312 (0.0018)	0.2427 (0.002)	0.0957 (0.001)	0.0500 (0.0006)
k=2/7	0.4272 (0.0046)	0.4450 (0.0019)	0.1901 (0.0023)	0.0777 (0.0011)	0.0420 (0.0007)
k=3/7	0.2276 (0.0037)	0.5604 (0.0017)	0.1275 (0.0022)	0.0549 (0.001)	0.0312 (0.0006)
Vulnerability estimates with logit models					
k=1/7	0.7380 (0.0039)	0.3333 (0.0018)	0.2460 (0.002)	0.0969 (0.001)	0.0517 (0.0007)
k=2/7	0.4379 (0.0045)	0.4455 (0.0018)	0.1951 (0.0023)	0.0799 (0.0011)	0.0442 (0.0007)
k=3/7	0.2349 (0.0037)	0.5598 (0.0018)	0.1315 (0.0022)	0.0570 (0.001)	0.0332 (0.0007)

Standard errors in parenthesis. H: Poverty headcount ratio; A: Poverty Intensity; M_0 : adjusted multidimensional poverty; vulnerability measures: V^0 , V^1 , V^2 .

E. Robustness Analysis

The proposed VMPI has been the subject of several decisions. In order to construct the multidimensional vulnerability measurements, we have made decisions about the indicator weights, the multidimensional poverty cut-off and the risk aversion parameters γ_m . Thus, to ensure that the VMPI is a robust indicator, we must compare if our results are consistent with those we would have obtained with alternative parameterizations. This is what is called submitting the measure to a robustness analysis. In order to carry out this analysis, we will compare our results with two alternative specifications of multidimensional cut-off, two alternative specifications of indicator weights and four alternative specifications of the risk aversion parameters γ_m .

The comparisons by different cut-offs were performed with $k = 1/7$, $k = 2/7$ and $k = 3/7$. The comparisons by weights correspond to the following versions:

(1) baseline scenario; (2) hot water and public security weights are decreased to $1/14$ and those of schooling and income are increased to $3/14$; (3) the weight of housing materials is increased to $1/14$ and the weights of people per room and housing tenure are decreased to $1/28$. Comparisons by risk aversion parameters were performed to the following γ_m values: 0.6, 0.7, 0.8, 0.9 and 1.

We compare the results of the measurement obtained in the 16 regions of Chile and we see if the order of the vulnerability measurement by regions remains similar when making changes in the specification of the critical parameters. We implemented the strongest possible comparison in terms of ordering random results. We made all possible pairwise comparisons of results between the regions of Chile under different parametrizations using inference tools that are equivalent to the second order stochastic dominance comparison criterion. Based on the hypothesis testing with regard to the change in parameters, a pairwise comparison is robust if the statistical order existing in the baseline scenarios remains in other scenarios. In keeping with Alkire et al. (2015, p.237), the statistical pairwise comparison of the adjusted incidence measures (M_0 in the case of poverty whose analogue is V^0 in the case of multidimensional vulnerability) using this recent described tool of statistical inference generates the same type of comparison as the second-order stochastic dominance criterion. This type of comparison may be too stringent and not hold in some cases. However, if the measurement is robust, it must keep the statistical order in most cases. Table 8 shows the ratio of robustness for the different comparisons, which is equal to the number of robustness pairwise comparisons divided by the total pairwise comparisons performed. As we see in the table, the ratio of robustness is high for the three executed robustness exercises.

TABLE 8—ROBUSTNESS ANALYSIS: PAIRWISE COMPARISONS BY REGION

	MPI	VMPI probit	VMPI logit
Different cut-off comparisons			
Possible pairwise comparisons	120	120	120
Robust pairwise comparisons	106	108	109
Ratio of robustness	0.88	0.90	0.91
Different weight comparisons			
Possible pairwise comparisons	120	120	120
Robust pairwise comparisons	101	96	97
Ratio of robustness	0.84	0.80	0.81
Different gamma coefficients comparisons			
Possible pairwise comparisons		120	120
Robust pairwise comparisons		110	111
Ratio of robustness		0.92	0.93

V. Concluding remarks

A model for measuring vulnerability to multidimensional poverty has been proposed in this paper. The introduced measurement is supported on an asymmetric mean-risk dominance criterion for the unidimensional identification of vulnerable individuals in each welfare dimension. Then, the AF method is used in the multidimensional identification stage and in the aggregation phase. The family of multidimensional vulnerability indicators developed here, belongs

to the AF class of multidimensional measures and therefore it is also associated to the FGT class of poverty indicators. Consequently, it shares with the AF family and with the FGT class, the fulfillment of a broad desirable axiomatic basis for poverty measurements.

This new vulnerability to multidimensional poverty index, the VMPI, is feasible to be estimated with cross-sectional data which constitutes a great advantage to be used in practice by policymakers jointly with the multidimensional poverty index (MPI).

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APPENDIX

TABLE A1—MISSING DATA SUMMARY

Indicator	Missing Values	Observations	Percent Missing
People per room	238	214,420	0.11%
Housing materials	0	214,420	0.00%
Housing tenure	0	214,420	0.00%
Hot water availability	0	214,420	0.00%
Public security	87	214,420	0.04%
Schooling	0	214,420	0.00%
Health	5	214,420	0.00%
Employment	0	214,420	0.00%
Income	0	214,420	0.00%