

Renegotiating Taxes and Political Risk in the Extractive Industries: The Effect of Progressive Tax Regimes

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PRELIMINARY DRAFT
August, 2012

Abstract

This paper explores how the renegotiation of taxes in the Natural Resources Industry, especially the introduction of *progressive taxation*, affects the *political risk* of the industry. Following the real options model developed by Schwartz and Trolle (2010), the *political risk* is reduced to an expropriation decision taken by the government modelled as the optimal exercise of an American-style option; whereas comparable *progressiveness in taxation* is achieved either using non-linear taxes or windfall taxes that are ex ante equivalent to a flat corporate profit tax. It was found that for an interval of initial market conditions—low spot commodity prices or futures contracts curves in backwardation—the introduction of an equivalent but more progressive tax regime decreased the political risk and the corresponding deadweight loss. However, when initial prices were too high or initial futures curves were in strong contango, the introduction of a progressive tax regime could significantly increase political risk. Finally, the model was calibrated with production data from the oil and copper industries.

JEL Classification: D81, G13, H25, H30, Q34, Q38.

Keywords: Nonrenewable resources, political risk, expropriation risk, taxation, real options, progressive taxation, equivalent taxation.

1 Introduction

The extraction of petroleum and minerals are the most important economic activity for many developing countries;¹ nevertheless, the positive externalities for the rest of the economy are limited given the fact that these activities are both capital intensive, usually located far from population centres, and in the case of developing countries, with limited capability to source

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¹cf. Boadway and Keen (2010) for a general review and Otto (2000) for the mining industry case.

locally. Therefore, the social benefit for a country of having foreign owned firms involved in the extraction of non-renewables is mostly associated with the taxes governments can collect.

The importance of these extractive activities generates huge incentives for host governments to change the tax conditions once investments have been sunk. Governments and firms know this, and as a consequence there is going to be underinvestment in presence of weak ownership rights;² in these cases, governments can try to negotiate with incumbent firms a tax regime that will be inherently **more stable** without sacrificing expected income. This paper shows the effect of the introduction of such equivalent tax regimes on the political risk, so that we are then able to know when a new and more progressive taxation regime can be of use in decreasing the perceived risk of investing in a given country.

A small aside is that this paper will focus solely on the operations that involve the extraction of non-renewable natural resources, as a one of the assumptions is that the production function is finite, therefore the term 'natural resources' is restricted to its use as 'non-renewable natural resources'.

1.1 Motivation

This paper thus wants to show how the incentives to renege from an existing tax regime in the natural resources industry can be modified by a new tax regime that can be mutually agreed by both the companies and the government. If that is the case, then it would be feasible to increase the overall social value of the natural resource operations by eliminating the deadweight loss associated with expropriations.

Dynamic inconsistency of taxes in the natural resources industry

Investment in the extraction of natural resources is characterised by a substantial upfront investment which then becomes largely immobilised at the site of operations. For this reason, the most important decisions for private firms are taken *ex ante*; as after the investment has been made they are left in a vulnerable position vis-à-vis the fiscal authority.

This paper aims to tackle one of the issues that arise in the design and implementation of taxation regimes for the natural resources industry, namely, how governments can deal with their *inability to commit themselves* to a previously agreed taxation environment. This is a major issue both for the developing world and for natural resources companies: there is a deadweight loss associated with lack of commitment, which in its turn implies that a discretionary policy decreases the benefit coming from natural resources exploitation.

Governments are keen to attract foreign investment to develop new natural resources project. In cases where there is no reputation for good governance it is not uncommon for the same government to offer guarantees of intertemporal tax stability. Nevertheless, if there are abrupt changes in the prices of the underlying commodity these guarantees increase the risks that the government defaults on its promise and changes the supposedly stable tax regime. Investors are able to foresee this, and will therefore demand higher rates of return in compensation.

Firms are therefore going to demand a higher rate of return to compensate for the increased perceived risk, thus altering the expected return of extractive operations for the host country.

²cf. Bohn and Deacon (2000).

This is why in such situations the inability to commit lowers the social benefit of the natural resources industry relative to the scenario where the possibility of a government reneging to its tax regulation is lower.

Hypothesis: Less political risk with more progressive taxation

The **hypothesis to be tested** in this paper is that in absence of taxation commitment a more robust and progressive taxation regime can be found without changing the total expected tax capture so that it significantly diminishes the risk of default from the state. The fact that these new tax regimes have the same expected discounted value would make firms indifferent between any of them ex ante in absence of political risk, so that any variation in the value of the operations that these different taxes generate can be attributed entirely to the effect of progressiveness in taxation.

The situation today

Stroebel and van Benthem (2010) analyse 2,466 oil contracts among different legislatures and find that tax capture is largely linear on the price of the commodity; that is, the tax rate is almost flat (i.e. not progressive). They argue that it might be the case that linear contracts are close to optimal, and therefore there are no incentives to deviate from the current taxation regime. Alternatively, they state that even in those cases where a more progressive taxation regime would be desirable, there might be signalling effects and cognitive costs that make transition into a more progressive tax regime too costly.

In this paper we argue that changing from linear to progressive tax regimes does not necessarily need to be that costly; a reform that introduces progressiveness in taxation in such a way that it is equivalent ex ante to the previous tax regime should be accepted, in principle, by both governments and companies. For this reason, the introduction of a family of equivalent tax regimes might be considered as a proxy for negotiating under non-threatening conditions.

1.2 Literature review

The analysis of taxation within the extraction of non-renewables is conditioned by the unique characteristics of this activity, the most obvious of these—exhaustibility—was first dealt with in the seminal article by Hotelling (1931). Boadway and Keen (2010) provide an exhaustive list of why taxation in the natural resources industry is of special concern, while Osmundsen (2010) provides reasons for society's low tolerance to profits in this industry.

Selecting a tax regime for this industry is hampered also by the asymmetry in available information; there is a problem of incentives which was analysed in an elegant two-stage model for efficient taxation of non-renewables by Osmundsen (1998). Nevertheless, that model assumes that governments are able to honour their tax promises; Engel and Fischer (2010) relax this assumption and build a closed model to optimise the taxation revenue from natural resources, assuming an exogenous 'expropriation function' to indicate when governments renege from their commitments, the optimal taxation regime thus obtained is similar to a resource rent taxation as introduced by Garnaut and Ross (1975), a very progressive taxation structure that

extracts more of the rent in high commodity price scenarios. Rigobón (2010) goes a step further and models the production response from producers if facing expropriation regime, albeit with an extremely simple and exogenous procedure to determine the expropriation decision. A more elaborate model for an *endogenous* expropriation decision is considered by Schwartz and Trolle (2010), which constitutes the basis for the expropriation decision model used here. The use of real options for decision models in a non-renewables context as was used in that paper is taken from the work initiated by Cortázar and Schwartz (1998) and from the valuation of risk as a real option by Mahajan (1990) among others. Finally, Stroebel and van Benthem (2010) analyse existing oil contracts in search of taxation regimes which vary with the underlying commodity price, only to discover that there is little evidence of tax regimes that try to capture additional rents when commodity prices increase. Both Stroebel and van Benthem (2010) and Johnston (2008) argue for the necessity to introduce 'progressive' elements to the taxation of natural resources: the effects that these same progressive elements have on the perceived political risk is what is going to be analysed in this paper.

1.3 About the structure of this paper

This paper takes the expropriation decision model by Schwartz and Trolle (2010) and expands it in order to assess the effects on political risk (as faced by natural resources firms) of varying progressiveness in taxation. Thus, the main modifications/extensions to the original expropriation decision model that are implemented are:

- The introduction of a family of equivalent tax regimes, taxes that could be in principle agreed upon by both the government and the extraction companies as a replacement for the current tax regime, and then observe the effect of varying progressiveness on political risk.
- The adjustment of the dynamic pricing process, so that the no-arbitrage condition on the prices of futures contracts verifies at every moment.
- And finally, using additional dynamic price models to represent the behaviour of different commodities.

These models, that value political risk in the natural resources industry are detailed in **Section 2**; where the dynamic price models for crude oil and copper are introduced together with two types of equivalent taxation of varying progressiveness, nonlinear and windfall. Finally, the section concludes with the introduction of a modified version of the expropriation decision model by Schwartz and Trolle (2010); which in this case allows for different tax regimes to be used. This section concludes with an outline of the methodology (the 'experimental setting') together with an overview of the most relevant results obtained, which is analysed in detail in the section that follows.

The results for the assessment of the expropriation risk for mines/oil fields under varying progressiveness are shown in **Section 3**. This section starts reviewing the initial conditions for

each Monte Carlo simulation—which will determine decisively the price trajectory given the no-arbitrage condition—for both copper and crude oil prices as reviewed in this paper. Then, results are shown for both commodities under nonlinear and windfall tax regimes. These results are finally analysed by the end of the section, where the departures for the initial hypothesis are explained.

Finally, **Section 4** concludes; a set of recommendations to successfully negotiate change is introduced together with some suggestions to extend the model further.

The results in a nutshell

The hypothesis as it was defined—that a progressive taxation regime lowers the political risk of natural resources operations—was contrasted with the results obtained from different tax regimes and initial conditions, the main observations being the following:

Progressive taxation does not always help. When either the initial spot prices or the shape of the futures contracts curve is above a certain threshold, the introduction of progressiveness in taxation while keeping equivalence is counterproductive. Governments become impatient and increase the rate of expropriation, while the value of this same expropriation decision decreases initially as it is taken on worse terms than it was done without progressive taxes.

Windfall vs. Nonlinear taxation regimes. Nonlinear tax regimes are riskier than comparable windfall tax regimes if highly progressive; however under some range of initial conditions, windfall tax regimes are subject to bigger discrete variations under modest changes in the progressiveness parameter.

The effect of different price models. Two different dynamic price models were used, one taken directly from Trolle and Schwartz (2009) to simulate oil prices, while the other was calibrated using copper price and futures data using the model by Cortázar and Schwartz (2003). Results from these models—both of them assume a no-arbitrage condition—showed differences sensitivity to progressiveness; though in both cases under sufficiently adverse initial conditions, political risk increased with progressiveness.

2 Valuing political risk using real options

In this section we introduce the methodology that was employed to evaluate how political risk varied in presence of a mutually agreed renegotiation of the tax regime. First we describe the basic assumptions behind the valuation model; then we introduce the concept of equivalence and progressiveness in a tax regime as a proxy for a mutually agreed tax reform in the natural resources industry. Finally, we describe the experimental setting and overview the results that are going to be detailed in next section.

2.1 The basic framework

This paper adapts the expropriation risk model by Schwartz and Trolle (2010), which employs a real options model to determine the dynamics of an expropriation decision in the natural resources industry as taken by a government. In this paper, though, this model is extended to see how this risk evolves after a new tax regime has been introduced.

Like in the seminal paper by Mahajan (1990), the expropriation decision is modelled as the exercise of an American-style option, where the payoff is given by the balance between the gains and costs of expropriating at any given time, and the early exercise of this option represents the expropriation decision itself. The advantage of representing the political risk through an American-style option is that it takes into account the fact that it may be on the best interest of a government to withhold expropriation—even if the payoff were positive at that time—due to a higher expected continuation value for the aforementioned option. However, unlike previous models, the resulting real option cannot be solved in a closed form, and therefore we use Monte Carlo methods to simulate price paths and value the exercise of this model. Numerical methods allow us to use more complex dynamic price models that attempt to better represent the reversion to the mean shown by commodity prices; different underlying price models can be compared without extensive modifications to the valuation framework introduced here.

The price models used in this paper are presented in detail on the appendix; one of them is parameterised for crude oil while the other represents the copper market, both of them generate stochastic spot prices and the associated futures contracts curve for each generated spot price. The common factor between these two models is that in both cases there is a no-arbitrage condition in the futures contracts market: Futures contracts are unbiased indicators of the market's expectation for the price of the commodity, so that by construction we have the following equivalence,

$$F(t_j, t_l) = E_{t_j}[S_{t_l}], \quad (42)$$

where $F(t_j, t_l)$ is the value at time t_j of the future contract maturing at time t_l , and $E_{t_j}[S_{t_l}]$ is the expectation at time t_j of the spot price of the commodity at time t_l .

To simulate crude oil spot and future prices, the data and parameters were taken from Trolle and Schwartz (2009)—the same price model used in the original expropriation risk model by Schwartz and Trolle (2010)—whereas for copper we used the model by Cortázar and Schwartz (2003), calibrating it with copper spot prices and futures from 2000 to 2010 with futures contracts expiring up to 120 months from the spot price date.

As with any numerical simulation, prices vary discretely between each period; in this case the complete time horizon of the simulation is divided into N steps, so that the time corresponding to the j step can be described as t_j , where $j \in \{1, \dots, N\}$ and $t_N = T$, with T the time horizon of the extraction process, so that each period t_j has a length of $\Delta t_j = T/N$.

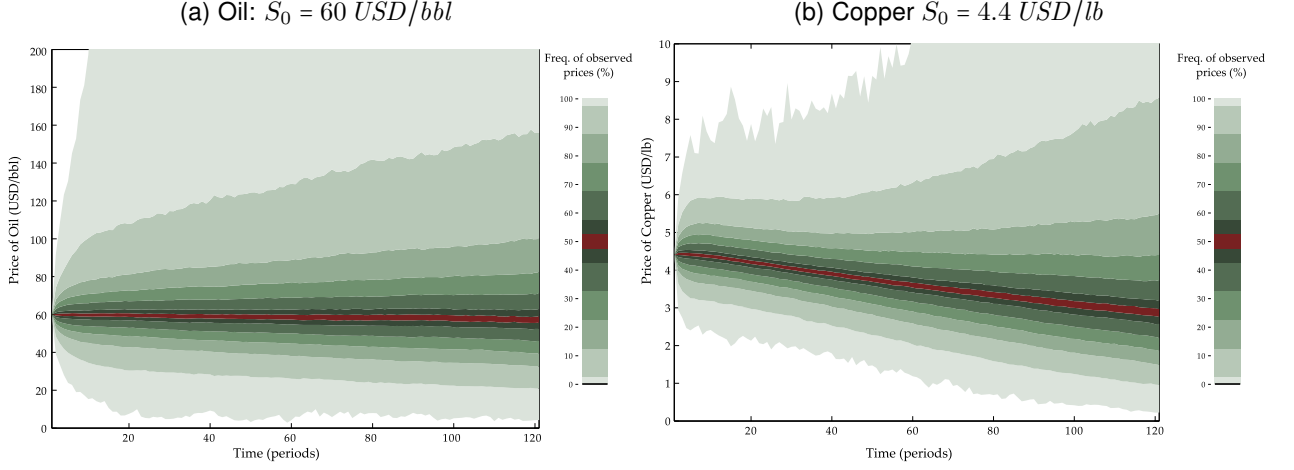
Therefore, for each commodity we obtain a series of spot prices and futures contracts of the form: S_{i,t_j} and $F_i(t_l, t_j)$ respectively, where $i \in \{1, \dots, M\}$ is one of the M independent paths used in simulation.³

In figure 1 below we can see how the distribution of spot prices evolves throughout the life

³Throughout this paper, and unless explicitly told otherwise, each Monte Carlo simulation was run using 10,000 paths with antithetic variate reduction.

of the mine/oil field. In this figure, as in the rest of the paper, we assume $T = 10 \text{ years}$ and $N = 120 \text{ periods}$.

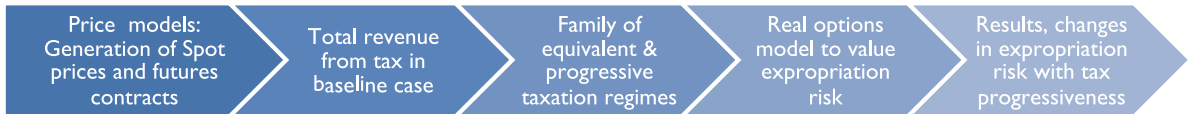
Figure 1: Frequency of observed prices using the dynamic oil and copper price models. Colours represent the distribution of the different price paths when $M = 10,000$.



As can be seen, both models show reversion to the mean: There is no exponential expansion of the distribution of prices, as it would be if we were simulating a typical random walk; this characteristic will later prove relevant for the conclusions of this model. We must not forget, either, that for each simulated spot price there will be a corresponding futures contracts curve with maturity up until the end of life for the mine or oil field.

In figure 2 we can see how we are going to proceed once these prices are generated; first we will estimate the total amount of tax that is going to be captured in a baseline case—a flat corporate profit tax as shown by Schwartz and Trolle (2010)—and then use that tax capture to construct a family of more progressive taxes that represent the outcome of an amicable negotiation. Finally, in section 3 we will assess the political risk of these new tax regimes and analyse the reasons behind the the variations of it.

Figure 2: The flow of information in this paper: Stochastic price models, equivalent and progressive taxes, and the valuation model for political risk.



2.2 Tax negotiation: Introducing equivalent tax regimes

The baseline tax structure for the operations of the mine or oilfield on which modifications are going to be introduced follows the structure presented by Schwartz and Trolle (2010): a simple flat rate tax on corporate profits, τ_{inc} . Where corporate profits are defined according to the following equation:

$$OpIncome_{i,t_n} = (S_{i,t_n} \cdot (1 - \tau_{roy}) - C_{firm}) \cdot Y, \quad (1)$$

Where for each time period t_n , $n \in \{1, \dots, N\}$, and each price path i where $i \in \{1, \dots, M\}$, $OpIncome_{i,t_j}$ is the corporate profit after royalties are paid, S_{i,t_j} is the price of the underlying commodity, τ_{roy} is a royalty tax on sales revenue, C_{firm} is the operational cost per unit of production and Y is the production rate for the commodity.

However, it must be noted that in this paper, and in order to isolate the effect of progressiveness in the different tax regimes, the royalty tax τ_{roy} is going to be kept equal to zero. For that reason, the total tax revenue at every price path i and time period t_n is going to be equal to:

$$Tax\ Revenue_{i,t_n} = \tau_{inc} \cdot OpIncome_{i,t_n}, \quad (2)$$

for each time period t_n and price path i ; and where τ_{inc} is the corporate tax rate.

Given the no-arbitrage condition, at every moment the government and the extracting companies can make an unbiased prediction of the future prices of the commodity, and therefore, they can also predict the expected total tax revenue from the exploitation of the mine/oil field if there is no risk of expropriation at all and if the tax rate is **kept constant** at its original value τ_{inc} . The fiscal authority may then wish to follow the advice mentioned in the literature and so choose to negotiate a new taxation regime that will be more progressive, in such a way that the perceived political risk decreases.

We can assume that in this case the industry and the government are able to negotiate a tax change that provides the same overall tax revenue for the government,⁴ and can therefore be voluntarily accepted by both parties. The possible outcomes of these negotiations are going to be called *equivalent tax regimes*, and will differ in the distribution of their payoffs as is going to be described below. But first we need a proper definition of what are we going to understand by *equivalent tax regime* and by *progressiveness in taxation*.

Definition 1. We can define an **equivalent tax regime** as one which provides a tax revenue flow that has the same expected discounted value than the baseline case. In this paper, this means any tax regime that provides—over the lifetime of the mine/oil field—a discounted value that is equivalent to the discounted tax revenue generated by a constant tax rate on profits, τ_{inc} .

Definition 2. A tax regime for a commodity is going to be **more progressive** than other if there is an underlying commodity price, S^* , so that for every price $S \geq S^*$ the average tax rate in the **more progressive** tax regime is **higher** than in the less progressive one.

Given these definitions, we can construct an unlimited number of tax regimes that are equivalent and more progressive than the baseline case. However, in this paper we limit ourselves to the study of two tax structures: a nonlinear tax rate which is contingent on the commodity price, and a windfall tax. The first of these is akin to what is proposed by Stroebe and van

⁴The initial debate for an appropriate level of taxation has been usually framed by this ex ante parity in tax revenue, as it is preferred that the companies voluntarily adopt the new regime. An example of this can be seen in the 2004-2005 Chilean discussion about mining royalties, as summarised in Congreso Nacional de Chile (2005).

Bentham (2010) and Johnston (2008); whereas the second is based on the results from Engel and Fischer (2010).

Both tax regimes replace the tax on the operational income, τ_{inc} , with a tax $T(S, k)$ where S is the underlying commodity price and k a parameter that varies according to the progressiveness of the tax regime; so that each value of k will represent an equivalent tax regime of varying progressiveness, in such a way that the total tax revenue becomes

$$Tax\ Revenue_{i,t_n} = T(S_{i,t_n}, K) \cdot OpIncome_{i,t_n} , \quad (3)$$

and the corresponding income for the companies is

$$After\ Tax\ Income_{i,t_n} = (1 - \tau_{inc}) \cdot OpIncome_{i,t_n} , \quad (4)$$

for every simulated price path i at period t_n .

A nonlinear tax subject to commodity prices

The first one of these tax regimes is one where the tax rate varies in direct proportion to the price of the underlying commodity. As it was mentioned before τ_{inc} is replaced by a tax of the form $T(S, k)$, which in this case would be:

$$T(S, k) = T(S_{i,t_n}, \delta) = \delta S_{i,t_n} + \tau_{base}(\delta) . \quad (5)$$

The parameter k is represented by δ , which modifies the progressiveness of the tax rate, as the expression $\delta \cdot S_{i,t_n}$ increases with the commodity price. In order to maintain the equivalence between with the baseline case, a flat base tax, $\tau_{base}(\delta)$ is introduced; which is defined as a function of δ in such a way that for every value of δ there is a single value of τ_{base} that makes the tax rate equivalent. The value of $\tau_{base}(\delta)$ will be given by the following equation:

$$\tau_{base}(\delta) = \frac{\sum_{i=1}^M \sum_{n=1}^N e^{-r(t_n-t_0)} OpIncome_{i,t_n} (\tau_{inc} - \delta S_{i,t_n})}{\sum_{i=1}^M \sum_{n=1}^N e^{-r(t_n-t_0)} OpIncome_{i,t_n}} , \quad (6)$$

So that for a given value of δ it is possible to find a value for τ_{base} that keeps both the government and the companies indifferent to a change in the tax regime from the baseline, τ_{inc} , into the nonlinear model as stated in equation (5).

It is possible then to generate a set of paired $(\delta, \tau_{base}(\delta))$ parameters so that the effect of modifying the progressiveness of taxation regimes can be assessed while keeping taxes equivalent. We limit here the analysis to a family of taxes bounded by the case where $\delta_0 = 0$, implying that there is a flat tax equal to $\tau_{base}(\delta_0) = \tau_{inc}$, and on the other extreme the case where δ increases so much that $\tau_{base} = 0$, we name this upper bound for δ as δ_{max} . Therefore, for all cases $\tau_{base}(\delta) \geq 0$ ⁵, while the generated parameters $(\delta, \tau_{base}(\delta))$ will evolve as follows:

⁵Thus avoiding situations where there would be a negative tax rate over operational income if the price were too low

$$\begin{array}{lll}
\delta_0 & \tau_{base}(\delta_0) & \rightarrow \delta_0 = 0, \text{ and } \tau_{base}(\delta_0) = \tau_{inc} \\
\delta_1 & \tau_{base}(\delta_1) & \rightarrow (\text{more progressiveness, with } \delta \uparrow \text{ and } \tau_{base} \downarrow) \\
\vdots & \vdots & \vdots \\
\delta_{max} & \tau_{base}(\delta_{max}) & \rightarrow \delta_{max} \geq \delta_n \forall n, \text{ and } \tau_{base}(\delta_{max}) = 0
\end{array} \tag{7}$$

Where the value of the upper bound for δ , δ_{max} , is defined by the equation:

$$\delta_{max} = \frac{\sum_{i=1}^M \sum_{n=1}^N e^{-r(t_n-t_0)} OpIncome_{i,t_n} \cdot \tau_{inc}}{\sum_{i=1}^M \sum_{n=1}^N e^{-r(t_n-t_0)} OpIncome_{i,t_n} \cdot S_{i,t_n}}, \tag{8}$$

in such a way that $\tau_{base}(\delta_{max}) = 0$, so that there is no fixed tax rate that is independent of the commodity price.

A windfall tax

An alternative way to define a more progressive taxation regime is with the institution of an equivalent windfall tax. The type of windfall tax assumed in this paper is one where there is an initial tax rate τ_{wf} until the price of the underlying commodity reaches a windfall price S_{wf} . When the price of the commodity is higher than S_{wf} , then all the additional income is taxed, and therefore equation (4) is modified so that after tax income becomes:

$$After\ Tax\ Income_{i,t_n}(\tau_{wf}, S_{wf}) = \begin{cases} (1 - \tau_{wf}) \cdot OpIncome_{i,t_n} & \text{if } S_{i,t_n} < S_{wf} , \\ ((1 - \tau_{wf}) \cdot OpIncome_{i,t_n}) - (S_{i,t_n} - S_{wf}) & \text{if } S_{i,t_n} \geq S_{wf} . \end{cases} \tag{9}$$

We can also formulate total tax capture using the expression $T(S, k)$ in equation (3), where the tax revenue is equal to

$$Tax\ Revenue_{i,t_n} = T(S_{i,t_n}, k) \cdot OpIncome_{i,t_n}, \tag{3}$$

so that the windfall tax can be stated as:

$$T(S, k) = T(S_{i,t_n}, \tau_{wf}) = \tau_{wf} + \mathbf{I}_{[S_{i,t_n} \geq S_{wf}(\tau_{wf})]} \cdot \left(1 - \frac{[S_{wf}(\tau_{wf})](1 - \tau_{roy}) - C_{firm}}{S_{i,t_n} - C_{firm}} \right) \cdot (1 - \tau_{wf}). \tag{10}$$

Where $\mathbf{I}_{[S_{i,t_n} \geq S_{wf}(\tau_{wf})]}$ is an indicator variable that takes the value 1 when $S_{i,t_n} \geq S_{wf}(\tau_{wf})$ and zero otherwise, and where S_{wf} is the windfall price, and τ_{wf} is the tax rate when prices are lower than the windfall commodity price.

In order to maintain the equivalence between this windfall tax and the baseline case, as the tax rate τ_{wf} decreases, the windfall price should also lower to compensate for foregone income when prices are lower. Therefore, the windfall commodity price is obtained as a function of τ_{wf} using the following identity:

$$S_{wf}(\tau_{wf}) = \left(\sum_{i=1}^M \sum_{n=1}^M e^{-r(t_n-t_0)} \left[\left(\mathbf{I}_{[S_{i,t_n} \geq S_{wf}(\tau_{wf})]} \cdot S_{i,t_n} (1 - \tau_{roy}) (1 - \tau_{wf}) \right) + \left((S_{i,t_n} (1 - \tau_{roy}) - C_{firm}) (\tau_{wf} - \tau_{inc}) \right) \right] \right) \cdot \left(\sum_{i=1}^M \sum_{n=1}^M e^{-r(t_n-t_0)} \mathbf{I}_{[S_{i,t_n} \geq S_{wf}(\tau_{wf})]} (1 - \tau_{roy}) (1 - \tau_{wf}) \right)^{-1}. \quad (11)$$

It must be noted that this identity has to be solved numerically, as $S_{wf}(\tau_{wf})$ is present implicitly at the right hand side of the equation in the indicator $\mathbf{I}_{[S_{i,t_n} \geq S_{wf}(\tau_{wf})]}$. Thus, using this equation is it possible to find a windfall price S_{wf} for any given initial tax rate τ_{wf} where $0 \geq \tau_{wf} \geq \tau_{inc}$; in such a way that both parties—the government and the extracting companies—are indifferent *ex ante* on whether to use the windfall tax or the baseline case with a tax rate equal to τ_{inc} .

As with the nonlinear tax, it is possible to generate a set of paired $(\tau_{wf}, S_{wf}(\tau_{wf}))$ parameters to affect the progressiveness of the taxation regime while keeping taxes equivalent. The parameters are bounded by the initial case where $\tau_{wf0} = \tau_{inc}$, and therefore the tax is equivalent to the baseline case, to the case where $\tau_{wf(min)} = 0$ and therefore $S_{wf}(\tau_{wf(min)})$ is the lowest value for the windfall spot price. The parameters that are generated according to this rule will be of the form:

$$\begin{array}{lll} \tau_{wf0} & S_{wf}(\tau_{wf0}) & \rightarrow \tau_{wf0} = \tau_{inc}, \text{ and } S_{wf}(\tau_{wf0}) = \infty \\ \tau_{wf1} & S_{wf}(\tau_{wf1}) & \rightarrow (\text{more progressiveness, with } \tau_{wf} \downarrow \text{ and } S_{wf}(\tau_{wf}) \downarrow) \\ \vdots & \vdots & \vdots \\ \tau_{wf(min)} & S_{wf}(\tau_{wf(min)}) & \rightarrow \tau_{wf(min)} = 0, \text{ and } S_{wf} \text{ has its lowest value} \end{array} \quad (12)$$

In the following paragraph we will see the difference between these two taxation regimes. and how the tax capture evolves with the underlying commodity price.

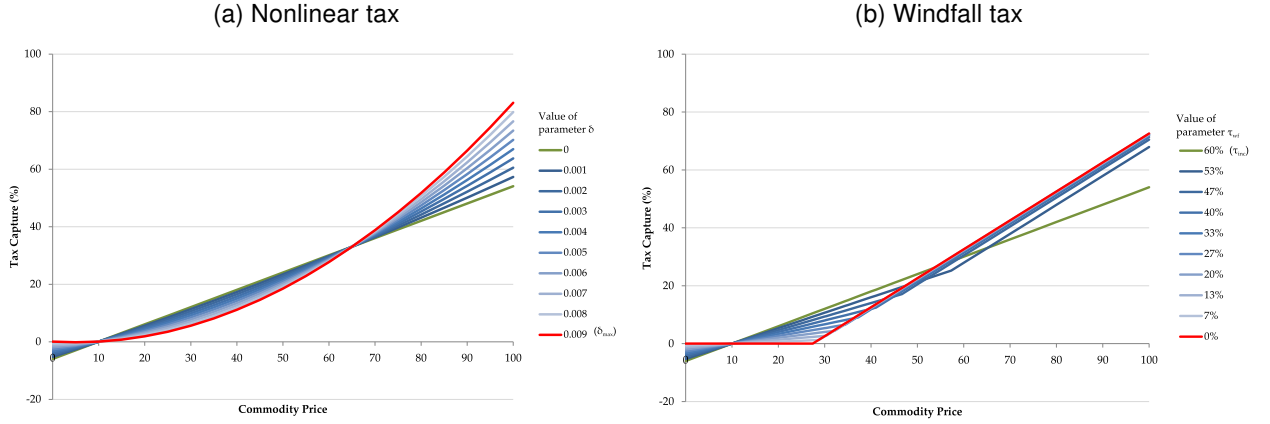
Nonlinear vs. windfall

To illustrate the difference between the initial baseline case and the equivalent progressive tax regimes it is useful to see how the amount of tax collected varies when the underlying commodity price changes. In figure 3, the two equivalent tax regimes are shown—nonlinear and windfall tax—for identical initial conditions;⁶ as progressiveness increases in both cases (a higher value of δ or a lower value for τ_{wf} in the case of windfall tax regime) the amount of collected tax increases with the higher price of the underlying commodity.

These tax regime examples were built using the oil price model. As could be seen in equations (6) and (11), to calculate the new tax regimes parameters we require to know the values of the baseline tax rate, $\tau_{inc} = 0.60$, and the operational cost of the firm, $C_{firm} = 10 \text{ USD/bbl}$, which in this case are the values used in the following expropriation model as well as in the original version by Schwartz and Trolle (2010). For this reason we see that there is in effect

⁶As tax regimes are equivalent, the value of the parameters for these new tax regimes will change if the initial conditions (basically, the spot price and the shape of the futures contracts curve) are different in such a way that the expected tax capture *ex ante* remains the same.

Figure 3: Tax capture with varying progressiveness under different commodity prices: Graphs made using the nonlinear and windfall equivalent taxes for the case of oil with initial spot price of $S_0 = 50$ and parameter $x_0 = 0$. The green line represents the baseline case, with a constant tax rate $\tau_{inc} = 60\%$, whereas the red line represents the maximum progressiveness for each tax regime.



a *negative tax* when prices are lower than 10 *USD/bbl*, where operational income is negative. This could be interpreted as a standard tax credit on losses, which does not affect results qualitatively.

Now that the equivalent taxation regimes are properly defined, we can examine how the political risk is going to be valued. The details of the valuation model and of the experimental setting are detailed below.

2.3 The expropriation model by Schwartz and Trolle (2010)

Up until here all calculations have not considered their political risk implications. To value how equivalent taxation regimes of varying progressiveness affect this risk we use a modified version of the model presented by Schwartz and Trolle (2010) (the baseline model); which reduces political risk of a natural resources project to the expropriation decision as taken by a *rational* and *risk-neutral* government. This expropriation decision is taken as the optimal exercise of an American-style option, the payoff at the exercise of this real option would leave the government with the full income stream from the natural resources project, this is balanced with the fact that extraction costs will be higher and that the government should pay a compensation to incumbent operators after expropriating the assets as well as additional reputational costs that will not be transferred as compensation to former operators.

In the baseline model, Schwartz and Trolle (2010) show us that there is an *increased* risk of expropriation if the initial spot price of the commodity is *high* or if the futures curve of the same commodity is into a strong *contango* at the initial time t_0 : both conditions imply the possibility of higher profits in the future, and therefore of a greater probability for the government to recoup the associated costs of exercising the option.

As was mentioned before, the baseline case has constant and flat corporate income tax equal to τ_{inc} , which is going to be contrasted with more progressive equivalent taxation regimes, so that it is possible to quantify the effect that a more progressive taxation can have in *reducing the*

expropriation risk of natural resources operations. The *intuition* behind this is that when prices for a commodity are higher, a higher tax capture will leave the expropriation option as a less attractive alternative. As has been mentioned before, this fact has been acknowledged in the model by Engel and Fischer (2010), who proposes a version of a windfall tax to extract most of the economic rent from producers, thus eliminating an—exogenous—risk of expropriation by dramatically increasing the marginal tax rate; this paper differs in that the risk of expropriation is endogenous, and that the amount of rent that can be extracted from producers is constrained by the fact that tax changes are equivalent to the baseline case as defined before.

The valuation model for political risk

According to this model, at each period t_j (with $t_j \in \{t_0 \dots t_N\}$), governments need to take a decision on whether to expropriate or not according to the perceived costs and gains of exercising an American-style real option on this decision to expropriate.

In order to value this option, the expected value of the operations is assessed using the spot prices and futures contracts of the underlying commodity: oil in the case of the dynamic price model by Trolle and Schwartz (2009) and copper in the case of the model by Cortázar and Schwartz (2003), these two dynamic price models are explained in detail in page 44.

As before, S_{i,t_j} is going to be the spot price of the commodity at time t_j in the i simulated path, while $F_i(t_j, t_l)$ is going to be the price for futures contracts in simulated path i at time t_j and maturing in t_l . The constant interest rate is going to be r , while τ_{roy} is the royalty rate over the sales, and the operational income tax is going to be stated in the form $T(S_{i,t_j}, k)$, which is either defined as the nonlinear tax (in page 9) or as the windfall tax (in page 10). Finally, we must consider that at each moment the government only knows the *expected* future value of the spot price, and if we consider the identity,

$$F(t_j, t_l) = E_{t_j}[S_{t_l}] , \quad (42)$$

used on the construction of the dynamic price models in the appendix on page 47, we can replace the operational income tax rate $T(S_{i,t_l}, k)$ with

$$T(F_i(t_j, t_l), k) , \quad (13)$$

for the value of the *expected* operational income tax rate at time t_l , that is, the future contracts are considered as unbiased predictors in a risk neutral setting. The cost of extracting one unit of the commodity is going to be C_{firm} for the firm and C_{gov} for the government; and it is assumed that $C_{firm} \leq C_{gov}$ so that expropriation increases the cost of production. The other costs of expropriation are the compensation paid to producers, assumed to be equal to K_1 per year of remaining operational life of the mine or oil field, and a reputational cost K_2 if the expropriation decision is taken. Finally, the extraction rate (or the production rate) is going to be Y_{t_j} an exogenous production function for each period t_j .

We can then estimate the value of futures contracts of different maturity at every moment for each simulated price path $i \in \{1, \dots, M\}$, where M is the total number of generated price paths. With these futures contracts we are able to estimate the instantaneous cost, $V_{cost}(i, t_j, k)$, and the gain, $V_{gain}(i, t_j)$, for the government of expropriating the operations in the price path i at time t_j , with a tax progressiveness given by factor k , according to equations (14) and (15).

$$V_{cost}(i, t_j, k) = \left[\sum_{n=j+1}^N e^{-r(t_n-t_j)} [(F_i(t_j, t_n)(1 - \tau_{roy}) - C_{firm}) \cdot T(F_i(t_j, t_n), k) + F_i(t_j, t_n)\tau_{roy}] Y_{t_n} \right] + K_1 \cdot (t_N - t_j) + K_2 \quad (14)$$

$$V_{gain}(i, t_j) = \sum_{n=j+1}^N e^{-r(t_n-t_j)} (F_i(t_j, t_n) - C_{gov}) Y_{t_n} \quad (15)$$

$V_{cost}(i, t_j, k)$ values the future tax income from the firm that is lost after the expropriation decision, as well as the reputational and compensation costs to be paid; while $V_{gain}(i, t_j)$ acknowledges the fact that after expropriation the government will receive the complete income stream from the operation. Thus, these two expressions are used to construct the payoff function for the exercise of the American-style expropriation option, this payoff should be:

$$\Pi(i, t_j, k) = \max(V_{gain}(i, t_j) - V_{cost}(i, t_j, k), 0), \quad (16)$$

for path i at time t_j and tax progressiveness given by parameter k .

In order to evaluate an American-style real option with this payoff function for any given value of k , we used the least squares Monte Carlo method (the LSM method) proposed by Longstaff and Schwartz (2001), which calculates the continuation value of the option, $E_{t_j}[\Pi(i, t_{j+1}, k)]$ (the unbiased expectation at time t_j of the value of the option at time t_{j+1}), using a linear regression at each step.^{7,8} What this means is that the owner of the option to expropriate (i.e. the government) tries to maximise its value by choosing an appropriate timing for its exercise. Once the real option model is solved using the LSM method, we know both the exercise time for every simulated path and the value of the option at time t_0 . Then, for each price path i there is going to be a period $Z_i \in \{1, \dots, N\}$ when the option is exercised; and if there is no exercise, $Z_i = N$, so that the time when the expropriation happens is going to be defined as t_{Z_i} . We can thus calculate the present value of the operations of the mine/oil field to the firm and to the government in adjusted to the risk of expropriation.

$$V_{firm}^{exp}(t_0, k) = \frac{1}{M} \sum_{i=1}^M \left(\left[\sum_{n=1}^{Z_i} e^{-r(t_n-t_0)} (S_{i,t_n}(1 - \tau_{roy}) - C_{firm})(1 - T(S_{i,t_n}, k)) Y_{t_n} \right] + e^{-r(t_{Z_i}-t_0)} K_1 \cdot (t_N - t_{Z_i}) \right) \quad (17)$$

⁷The option valuation algorithm was adapted from an American-style put option valuation that appears in Brandimarte (2006).

⁸At the period before operations cease, t_{N-1} , there's no uncertainty with respect to the value of the option—its continuation value is zero—so at that moment the option is valued at the value of the payoff function $\Pi(i, t_{N-1}, k)$.

$$V_{gov}^{exp}(t_0, k) = \frac{1}{M} \sum_{i=1}^M \left(\left[\sum_{n=1}^{Z_i} e^{-r(t_n-t_0)} [(S_{i,t_n}(1-\tau_{roy}) - C_{firm}) \cdot T(S_{i,t_n}, k) + S_{i,t_n}\tau_{roy}] Y_{t_n} \right] - e^{-r(t_{Z_i}-t_0)} (K_1 \cdot (t_N - t_{Z_i}) + K_2 \cdot \mathbf{I}_{[t_N > t_{Z_i}]}) + \sum_{n=Z_i+1}^N e^{-r(t_n-t_0)} (S_{i,t_n} - C_{gov}) Y_{t_n} \right) \quad (18)$$

Where for a given tax regime with progressiveness given by parameter k , $V_{firm}^{exp}(t_0, k)$ is the value of the operations for the firm in presence of expropriation, and $V_{gov}^{exp}(t_0, k)$ the corresponding value to the government; the indicator variable $\mathbf{I}_{[t_N > t_{Z_i}]}$ takes the value 1 if $t_N > t_{Z_i}$ and **zero** otherwise. The expressions in equations (17) and (18) should be compared to the value of the operations in absence of expropriation risk, V_{firm} and V_{gov} as defined below.

$$V_{firm}(t_0, k) = \frac{1}{M} \sum_{i=1}^M \left(\sum_{n=1}^N e^{-r(t_n-t_0)} (S_{i,t_n}(1-\tau_{roy}) - C_{firm}) (1 - T(S_{i,t_n}, k)) Y_{t_n} \right) \quad (19)$$

$$V_{gov}(t_0, k) = \frac{1}{M} \sum_{i=1}^M \left(\sum_{n=1}^N e^{-r(t_n-t_0)} [(S_{i,t_n}(1-\tau_{roy}) - C_{firm}) \cdot T(S_{i,t_n}, k) + S_{i,t_n}\tau_{roy}] Y_{t_n} \right) \quad (20)$$

Equations (19) and (20) are defined so that $V_{firm} = V_{firm}^{exp}$ and $V_{gov} = V_{gov}^{exp}$ in absence of expropriation risk, and $V_{firm} \geq V_{firm}^{exp}$ and $V_{gov} \leq V_{gov}^{exp}$ more generally. The presence of reputational costs, K_2 , generates a deadweight loss, according to the following function:

$$DWL(t_0, k) = (V_{gov}(t_0, k) + V_{firm}(t_0, k)) - (V_{gov}^{exp}(t_0, k) + V_{firm}^{exp}(t_0, k)). \quad (21)$$

It is also going to be of interest to look at the variation of the *value of the option to expropriate* at the initial time t_0 , $P(t_0, k)$, this will be equal to the average of the sum of the value of discounted payoffs at the exercise time, so that the ex ante value of the option will be

$$P(t_0, k) = \frac{1}{M} \sum_{i=1}^M e^{-r(t_{Z_i})} \cdot (1 - \mathbf{I}_{[t_N > t_{Z_i}]}) \cdot \Pi(i, t_{Z_i}, k), \quad (22)$$

where again, $\mathbf{I}_{[t_N > t_{Z_i}]}$ is an indicator variable that takes the value 1 if $t_N > t_{Z_i}$ and **zero** otherwise, and $\Pi(i, t_{Z_i}, k)$ is the value of the payoff function in equation (16).

Throughout this paper we refer to the total probability of expropriation which is going to be valued as the total amount of expropriation decisions that take place with respect to the total number of simulated paths, so that we have the value:

$$Total\ Prob.\ of\ Expropriation = \frac{1}{M} \sum_{i=1}^M \mathbf{I}_{[t_N > t_{Z_i}]} \quad (23)$$

Or the average number of times that the option is exercised, that is, whenever t_{Z_i} is smaller than the latest simulated period t_N . With these key indicators in mind we can now proceed to the actual implementation below.

2.4 Experimental setting

As was explained on page 8, equivalent tax regimes of varying progressiveness were introduced to account for the effect of a negotiation that may be agreed by both parties on the political stability of the regulatory framework. These equivalent taxes, either nonlinear or windfall, were arranged to show increased progressiveness⁹ so that the valuation of expropriation functions presented in the previous section can be of comparable nature.

Thus, while in Schwartz and Trolle (2010) the authors analysed how the political risk varied with different initial conditions for the dynamic price model for oil, or for static parameters such as K_1 or K_2 , in this paper we focus on how the expropriation risk varies with a more progressive taxation regime, that is, how does the variation of the parameter k affect the risk of the operations. The different initial conditions reviewed in this paper are an extension of the ones presented in Schwartz and Trolle (2010) and are detailed below.

In the original paper, the dynamics of the behaviour of commodity prices and of the related futures contracts are given by the single stochastic volatility model shown in Trolle and Schwartz (2009), it corresponds no-arbitrage model that considers an unspanned stochastic volatility and as such requires for its calibration the value of spot prices, futures curves and associated options from a complete options market. The calibration parameters found in table 1 of the paper by Trolle and Schwartz (2009)¹⁰ were used here—requiring data of crude oil spot price, futures and options traded on NYMEX—so that the results that we show here for oil production are comparable with those obtained by Schwartz and Trolle (2010), with an equivalent baseline case ($k = \delta_0 = 0$ for nonlinear taxes or $k = \tau_{wf0} = \tau_{inc}$ for the windfall tax).

In addition to the dynamic price for oil used by Schwartz and Trolle (2010), a copper price model was introduced using the framework presented by Cortázar and Schwartz (2003), assuming spanned volatility and mean reversal for the long term price of the commodity. This model was preferred over the one by Trolle and Schwartz (2009) for the copper case as it does not require a deep options market, but can nevertheless represent the dynamics of both the spot price and of futures contracts. To calibrate the model, futures and spot prices were used; in this case we used data for copper prices available from Bloomberg (3,375 observations from 23 June 1997 to 17 November 2010) both for the spot price and for futures contracts of up to 10 years.¹¹

Parameters

We also required to assign values for the multiple parameters in the expropriation valuation functions, such as the cost of exploitation by the incumbent company C_{firm} and the cost by the government C_{gov} . We relied, on one side, on the values used by Schwartz and Trolle (2010) for oil exploitation, while for the second set of simulations we relied on cost and operational data that was available from the Chilean copper commission (COCHILCO),¹² and for the tax

⁹Using taxation of the form $T(S, k)$, the progressiveness parameter increases from $k = \delta_0 = 0$ to $k = \delta_{max}$ for nonlinear taxes and decreases from $\tau_{wf} = \tau_{inc}$ to $\tau_{wf} = 0$ in the case of the nonlinear tax

¹⁰These are shown again in the first column of table 5 on page 47.

¹¹More information about setting these parameters is available on page 46 of the appendix.

¹²Cf. Titelman Goren (2007) for comparative data of the costs of private and public companies' extraction costs.

rate, on the information provided by Otto (2000) for the Chilean case. A summary of the values assigned to production and expropriation parameters can be seen in table 1 below.

Table 1: Comparison of parameters used for the simulation of expropriation models.

Constant parameters in the expropriation model			
Trolle and Schwartz (2009) Crude oil		Cortázar and Schwartz (2003) Copper	
τ_{roy}	0.00	τ_{roy}	0.00
τ_{inc}	0.60	τ_{inc}	0.35
C_{firm}	10	C_{firm}	1
C_{gov}	15	C_{gov}	1.7
Y_{t_n}	10/12	Y_{t_n}	10/12
T	10	T	10
N	120	N	120
K_1	50	K_1	3
K_2	1,000	K_2	60

Y_{t_n} , the production per period of time, $\Delta t = T/N$, is assumed to be constant unless told otherwise. T is the time horizon for the operations (in years); N is the number of discrete steps in the simulation process; K_1 , is the yearly compensation over the remaining lifetime of mine/oil field paid to the producers if expropriated. Finally, K_2 is the reputational cost that a country will endure if it exercises the expropriation option.

Initial Conditions

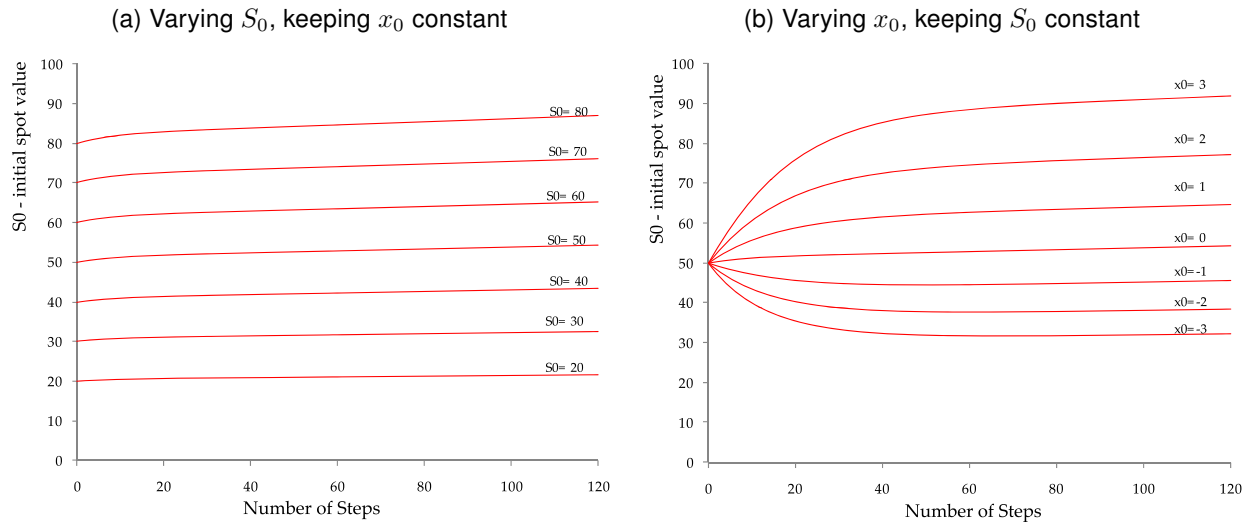
The construction of the dynamic price models for both oil and copper relied on the no-arbitrage condition, by which the unbiased expectation of the future spot price of the commodity at any given time is given by the price of the futures contract at that same moment, according to the identity,

$$F(t_j, t_l) = E_{t_j}[S_{t_l}], \quad (42)$$

so that the expectation formed at time t_j of how prices are going to be at time t_l remains equivalent to the corresponding future contract valued at t_j and maturing at time t_l . Therefore, not only the initial spot prices, but also the shape of the futures contracts curve is going to determine the expectations that both governments and firms have. These are going to change from period to period as the futures contracts change, but nevertheless the price expectation that was formed at time t_0 remains unbiased due to the construction of the price model.

For this reason we follow the methodology employed by Schwartz and Trolle (2010), that is, how the political risk—as defined by the previous valuation equations—changes given a set of initial conditions, as well as how they respond to a variation on the tax structure by modifying the tax progressiveness parameters. The changes to the initial conditions are detailed below.

Figure 4: Futures curves for oil prices using the Trolle and Schwartz (2009) model, varying price parameters S_0 and x_0 ; each step represent a period t_j measured in months.



The case of Oil Price: In the Schwartz and Trolle (2010) paper, both the initial spot price S_0 and the shape of the initial futures contracts curve were modified. We used the same approach in this paper to see how the effect of the initial conditions influenced political risk in presence of more progressive taxes, so that either the the shape of the futures contracts curve was left unmodified, by setting the parameter $x_0 = 0$,¹³ while changing the initial spot price of oil.¹⁴ Additionally, the effect of the shape of the futures contracts curve was determined by keeping the initial spot price, S_0 constant at 50 USD/bbl and modifying the vale of the parameter x_0 . How these initial conditions vary is shown in figure 4.

Copper values Initial conditions for the dynamic price model of copper were set analogously to the case of oil. First, the initial spot price, S_0 , was changed leaving the futures contract curve without modifications. Then, this curve was modified changing the parameter $\hat{v}_0$ in the model by Cortázar and Schwartz (2003).¹⁵ Finally, and taking into account that the Chilean Budget Commission¹⁶ forecasts a long-term price for copper (DIPRES, 2010), a third series of futures curves were created with different initial spot prices S_0 so that the long term price, after 10 years, was of 2.59 USD/lb , the long term outlook. All these initial conditions are shown in figure 5.

An overview of the results is presented here below while the complete results are available on

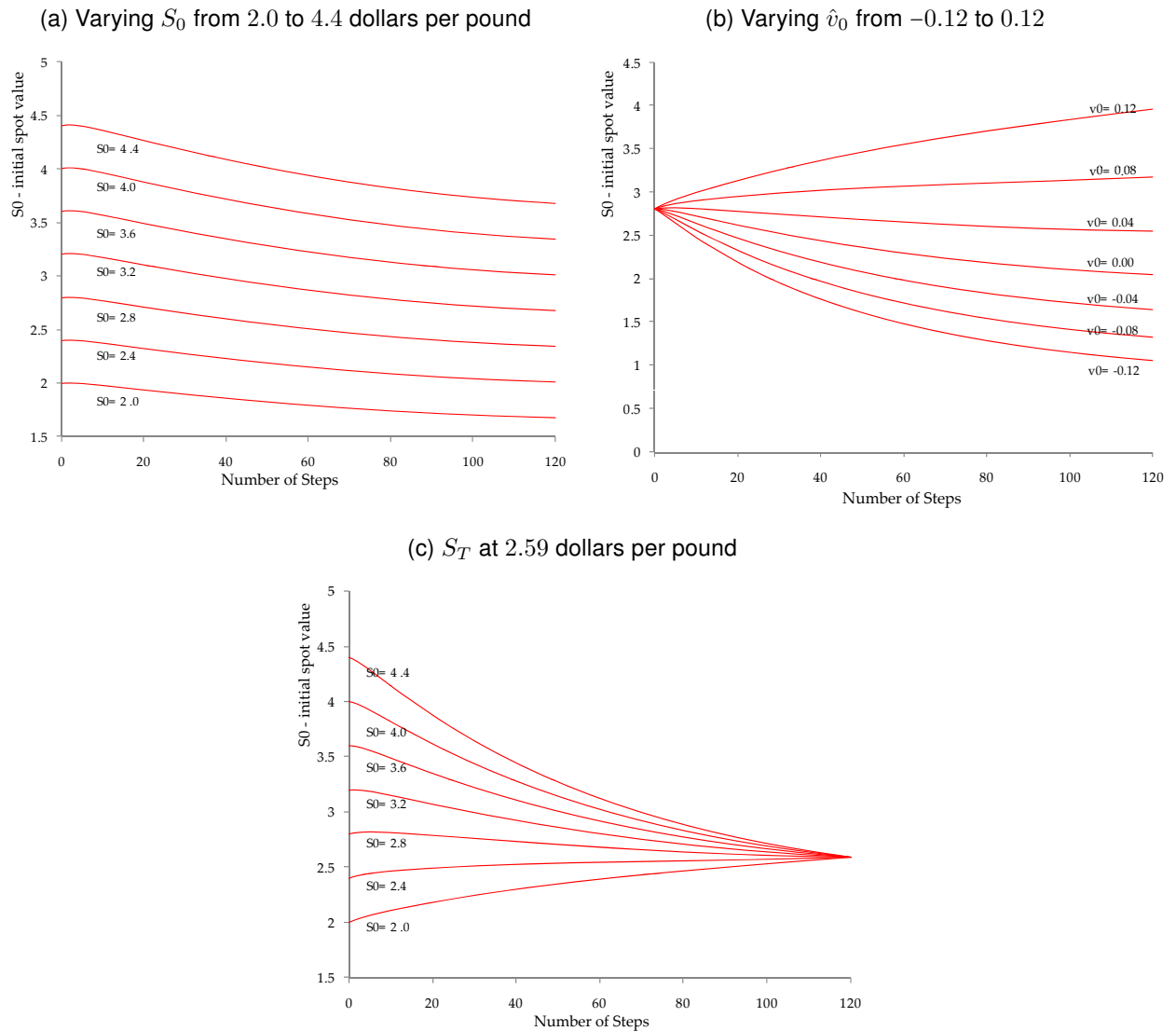
¹³The parameter x_0 defines the shape of the futures contracts curve in the Trolle and Schwartz (2009) dynamic price model, a positive value of x_0 implies a curve in strong contango while a negative value implies backwardation, as seen in figure 4b.

¹⁴The prices of oil selected for the dynamic price model in this paper are on the vicinity of 50 USD/bbl , to acknowledge the real prices used in the calibration of the model as shown in Trolle and Schwartz (2009).

¹⁵As with x_0 in the oil price model, a negative value implies backwardation while a large positive $\hat{v}_0$ means that the futures contracts curve is in contango.

¹⁶This is the *Dirección de Presupuestos*, DIPRES, Chile's budgetary office depending from the Ministry of Finance

Figure 5: Futures curves for copper prices using the Cortázar and Schwartz (2003) model, varying price parameters S_0 , $\hat{v}_0$ and finally varying both in order to make S_T equivalent to long term price of copper of 2.59 dollars per pound; each step represent a period t_j measured in months.



Section 3.

An overview of the findings

After running the political risk valuation model under different initial conditions and tax regimes, the results were compared with the ‘intuition’, the prior assumptions that motivated this research, and then an analysis was made to explain the resulting differences; below we can see a summary of what is covered in detail the following section.

The intuition: The working hypothesis made before the model was implemented was that a more progressive taxation regime would decrease the political risk of a natural resources

operation for an equivalent tax regime. With progressiveness in taxation as defined before in page 8, and with political risk defined solely as the exercise of the government's decision to expropriate the extractive operations.

Results: The intuition was confirmed to some extent: If the initial prices of the underlying commodity are low or if the futures curve is in strong backwardation we see that every single measure of risk decreases with the introduction of a more progressive tax regime; this is a Pareto optimum solution, as both the firm and the government see the value of their operations increase as the deadweight loss from expropriation recedes. Nevertheless, if prices are high enough, or if the futures curve is in *strong contango* we can see that the introduction of a more progressive taxation regime actually *increases* the risk of expropriation, in some cases reaching an almost certain and immediate take over by the government as soon as the new, more progressive, taxation regime is introduced.

Moreover, there were differences in the response to progressiveness in taxation between the different tax regimes: on one hand, the introduction of a nonlinear tax showed that under unfavourable initial conditions political risk increased gradually in tandem with progressiveness; whereas in the case of windfall taxes, there was a jump in risk with the introduction of progressiveness that could be eliminated if the degree of progressiveness increased.

Finally, when comparing oil and copper price models; there was an increase in the overall risk of expropriation for both commodities if the initial price was high enough or if the futures curve was in strong contango. However, the effect was more pronounced in the case of the oil price model than in the copper case, it is conjectured that the higher sensitivity to the progressiveness shown by the oil price model can be attributed to the stronger reversion to the mean of the copper price model together with the differences in volatility between both price models.

Analysis: The results are analysed in detail at the end of next section; where the *timing* of the expropriation decisions is considered. One example is figure 17 on page 34, which shows us the difference between the timing of expropriation that takes place on low risk environments—most expropriation taking place on the second or third year of operations—with the surge in expropriation that takes place in riskier environments as progressiveness in tax increases, with almost all the expropriation decisions taken immediately. The increase on the expropriation risk when progressiveness is higher is counterintuitive in the sense that the value of the total payoff function, $\Pi(i, t_j, k)$,¹⁷ decreases as predicted initially (and intuitively), however the expropriation decision is taken anyway because there are fewer prospects of expropriating profitably in the future.

Therefore, governments expropriate with higher progressiveness if prices are sufficiently large due to the fact that a decrease in the payoff function decreases the ability of the government to wait for further price increases; in such a way that governments become more *impatient* and for that reason the expropriation decision is taken in less favourable conditions. This is clearly shown when we see the evolution of an impatience coefficient

¹⁷As defined on page 14.

at the end of next section: governments expropriate a higher proportion of positive payoff functions when tax progressiveness increases.

3 Results, political risk and tax structure

In this section we present the results obtained from running the expropriation model with the two sets of equivalent taxation regimes defined previously, together with an analysis of the reasons behind the departure from the ‘intuition’ that motivated this research.

3.1 The parameters for the initial conditions

As has been mentioned before, in order to see how tax progressiveness affected the political risk of a natural resources project we changed the initial price and futures contracts conditions. A summary of the analysed cases can be seen in table 2 below, where every initial condition is identified with a letter from **A** to **K**.

Table 2: Summary of modified price model parameters to generate sets of initial conditions for simulation.

	Trolle and Schwartz (2009) Crude oil				Cortázar and Schwartz (2003) Copper			
	# [†]	Varying	Range	Keeping Constant	# [†]	Varying	Range	Keeping Constant
Nonlinear	A	S_0	47 to 60	$x_0 = 0$	C	S_0	2.0 to 4.4	$\hat{v}_0 = 0$
	B	x_0	-0.5 to 1.5	$S_0 = 50$	D	$\hat{v}_0$	-0.12 to 0.12	$S_0 = 2.8$
					E	S_0	2.0 to 4.4	$S_T = 2.59$
Windfall	F	S_0	47 to 60	$x_0 = 0$	I	S_0	2.0 to 4.4	$\hat{v}_0 = 0$
	G[‡]	S_0	61 to 74	$x_0 = 0$	J	$\hat{v}_0$	-0.12 to 0.12	$S_0 = 2.8$
	H	x_0	-0.5 to 2.0	$S_0 = 50$	K	S_0	2.0 to 4.4	$S_T = 2.59$

[†] Identification code for each simulated initial condition.

[‡] There is an additional run with an increased initial spot price, S_0 , given the robustness shown by the windfall tax regime.

Thus, for every commodity and every equivalent tax regime there is going to be a variation both in the initial spot price, S_0 , and in the shape of the futures contracts curve. As the equivalent windfall tax regime proved to be more robust, we had to analyse an extended range of initial spot prices to see how the results from the two tax regimes were comparable.

3.2 Intuition and results: Nonlinear progressive taxes

As could be seen previously in figure 3a on page 12, as the parameter δ increases in the non-linear tax regime the tax capture changes *smoothly* to a more progressive regime. As a result, political risk¹⁸ varies gradually as the progressiveness increases, in a direction determined by the initial conditions. The original ‘intuition’ was that a more progressive tax regime would minimise the political risk due to a higher tax capture when the underlying commodity price was

¹⁸Or probability of expropriation as defined in equation (23) on page 15.

higher, in the nonlinear tax case, this meant a lower risk of expropriation when the parameter δ has higher; this intuition can be contrasted with the results obtained for each commodity under variations of this tax regime which are presented below.

Oil price model under nonlinear taxes

When we analysed **how political risk varied with initial spot prices** we found that the more interesting results were in the region between $S_0 = 47$ and $S_0 = 60$. For this reason, here we show the results from that restricted interval of prices, which can then be extrapolated to the rest of the range without loss of generality.¹⁹

To acknowledge the increased progressiveness in taxation we chose a set of parameters δ_i as defined in page 9, where $\delta_0 = 0$ was equivalent to the baseline flat corporate tax rate, and δ_{max} is the case where $\tau_{base} = 0$, the most progressive tax that can be attained maintaining equivalence and avoiding negative tax rates.

The results thus obtained can be seen in figure 6; where the results for the political risk under different spot price conditions and progressiveness are shown for the nonlinear tax case. When prices are low, in this precise setting when S_0 is smaller or equal than 53, the *intuition* works and both the risk of expropriation and the deadweight loss decrease with a more progressive tax regime.

However, if the initial spot price is too high what we observe is that the risk of expropriation not only does not decrease with a more progressive taxation regime, but that actually this risk *increases greatly*, reaching almost certainty of expropriation when δ_i approaches δ_{max} . This result goes completely against the intuition, and the reasons for it to happen are explained in point 3.4 below.

There is also another interesting result if we look at the value of the price of the expropriation option as it is done in figure 7c, there we can see that even for higher spot prices, the value of the expropriation option *decreases* initially if we increase δ_i . That is, even though a government expropriates more in the case of high spot prices and higher δ_i what happens is that the value of the expropriation option is lower given that the conditions in which expropriation takes place are deteriorated.

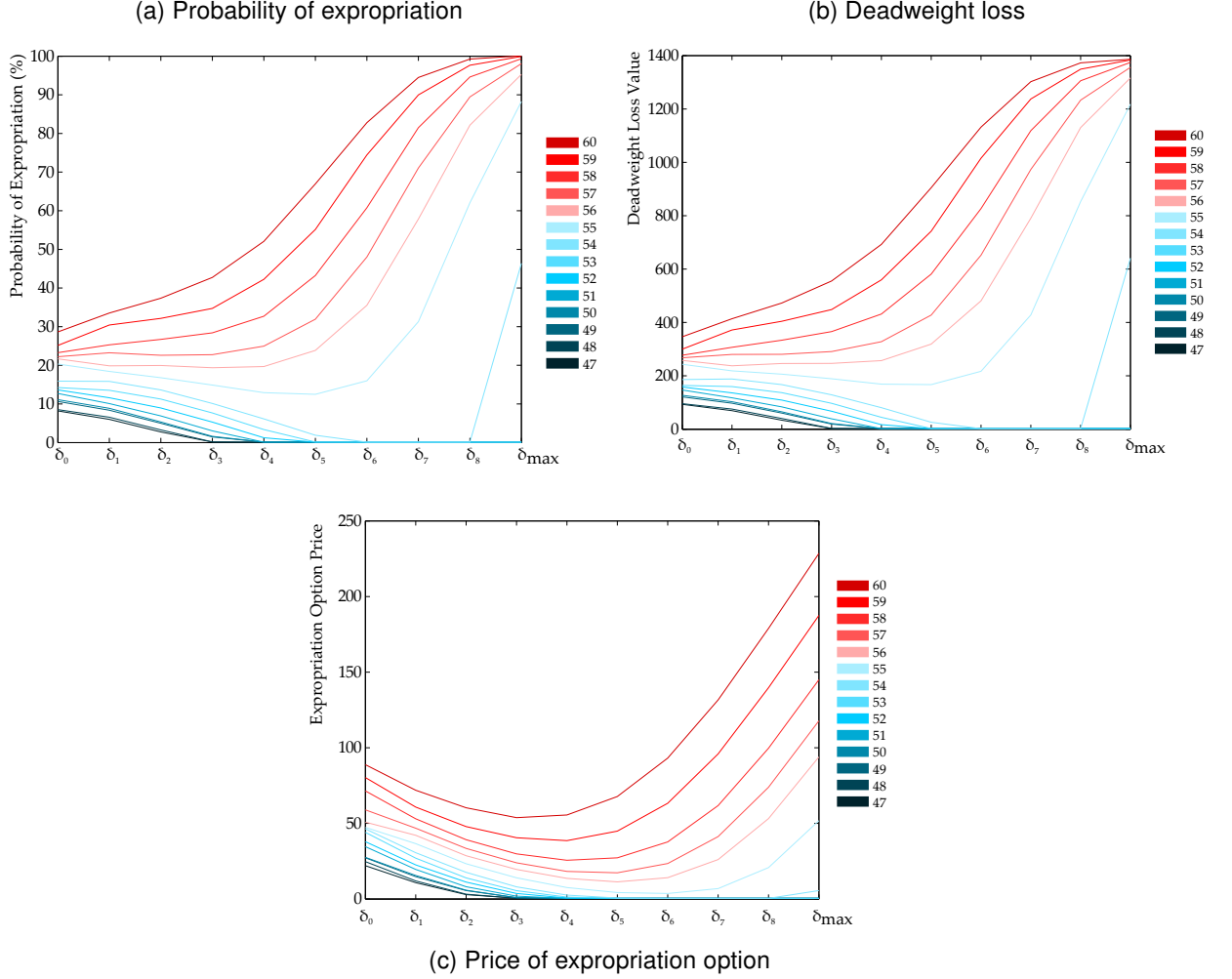
By construction of the model, there is a strong correlation between the expropriation decision and the deadweight loss. As we could see in the definition of the expropriation value for the government, V_{gov}^{exp} in equation (18), there is a single reputational cost, K_2 , that is not transferred to the companies and therefore is the source of the deadweight loss. As this correlation is kept throughout the studied cases, we will limit the analysis to the probability of expropriation and the price of the expropriation option from here onwards.²⁰

For the initial condition described as case **B** in table 2, that is, in order to see the effects that **the variation of the futures contracts curve has on the overall risk of expropriation for the nonlinear tax regime**, we analysed the expropriation model with varying initial conditions where the spot price remained constant at $S_0 = 50$ while the shape of the futures curve changed varying the parameter x_0 from $x_0 = -0.5$ (slight backwardation) to $x_0 = 1.5$ (contango) as it was

¹⁹This is case **A** as appears in table 2.

²⁰Equations (23) and (22) respectively.

Figure 6: Evolution of expropriation parameters (total probability, deadweight loss and price of the option) in the case of oil exploitation with increased progressiveness (given by δ_i) in nonlinear taxes with different initial spot prices S_0 , which vary according to the colour scale at the right of each figure (Case A in table 2).



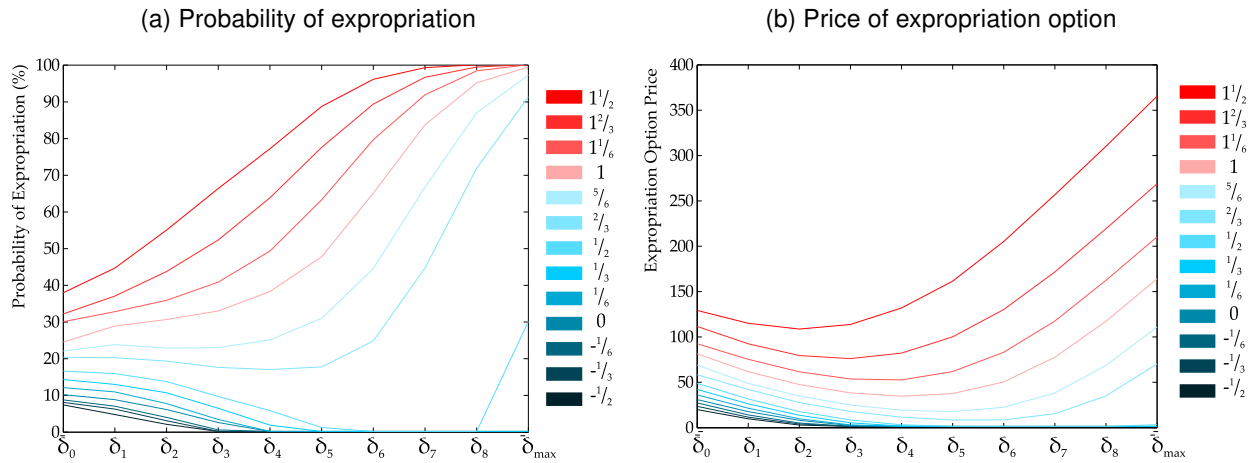
shown in figure 4b on page 18.

As before, the progressiveness of the tax regime was modified by increasing the parameter δ_i of the nonlinear taxation regime.

For this initial condition, the results for the expropriation risk and the value of the option for the government are shown in figure 7 below, where we can observe that, similarly to what was noted in the case where S_0 changed, an increase in the values of δ_i decreases the risk of expropriation when the prices are in backwardation or when they are in slight contango, however when x_0 is large enough the prospect of higher prices in the future increases the risk of expropriation.

What is interesting is that in this case the government is not responding to current high prices, but to future high prices that can be expected from the unbiased nature of futures contracts pricing. Moreover, the price of the expropriation option *falls* initially as δ increases, even in those cases where there is more expropriation (i.e. when x_0 is higher) this is due to the same cause that was seen before, that when δ_i increases, the value of the payoff function decreases

Figure 7: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of oil exploitation with increased progressiveness (given by δ_i) in a nonlinear tax regime with different initial shapes of the futures curves obtained by varying parameter x_0 which changes according to the colour scale at the right of each figure (Case **B** in table 2).



and the expropriation takes place in worst conditions.

Copper price model under nonlinear taxes

As has already been discussed, in order to generate price paths for the behaviour of copper prices, the model by Cortázar and Schwartz (2003) was selected due to its no-arbitrage property, to the fact that it had long term reversion to the mean, and given that the necessary parameters could be obtained from the information set that was available. This price model has also non-stochastic volatility, and so it is relevant to see if the results obtained in the case of crude oil apply also for this commodity.

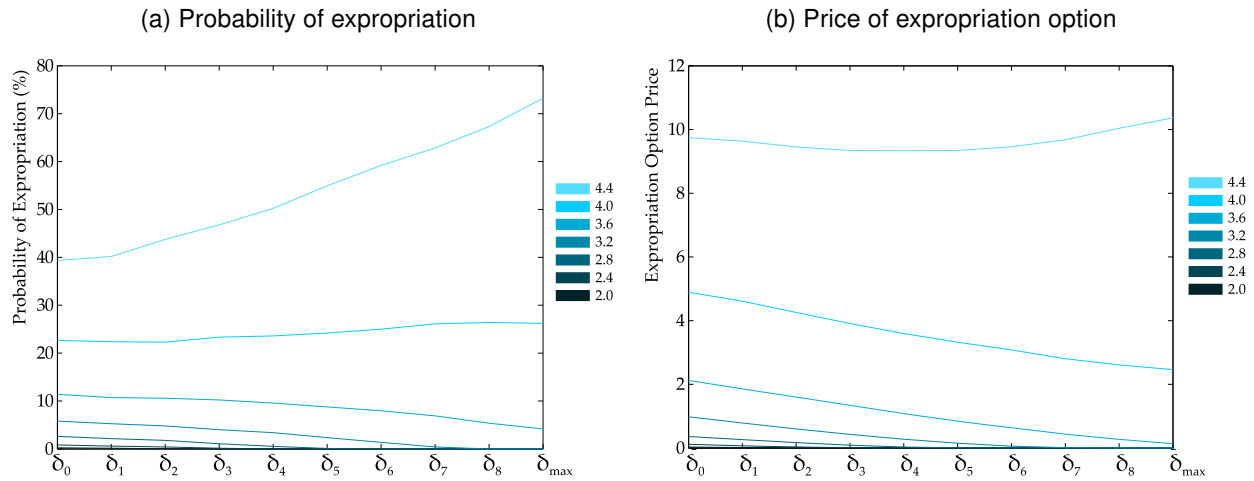
If results were comparable, then we would be able to extrapolate further results from one price model to the other, what really interested us then is whether we would still see an increase on the expropriation risk for high initial spot prices or for price curves in strong contango. Additionally, we observed the results for different initial conditions when there was a consensus over the long term price.²¹

As in the previous cases, we analysed how political risk varied with **different initial spot prices** ranging from $S_0 = 2.0$ to $S_0 = 4.4$ USD/lb for different values of δ_i (case **C** in table 2). The results for the total expropriation risk and for the price of the option value are shown in figure 8.

As it is shown in the initial conditions that apply to these results and that can be seen in figure 5a on page 19, when $\hat{v}_0 = 0$ the different initial futures curves in the Cortázar and Schwartz (2003) model are in backwardation, which according to what we just saw in the oil production case, makes expropriation less likely. For this reason the counterintuitive result of

²¹Here, long term is defined as T the time horizon for the exploitation and depletion of the mine or oil field, where $T = 10$ years.

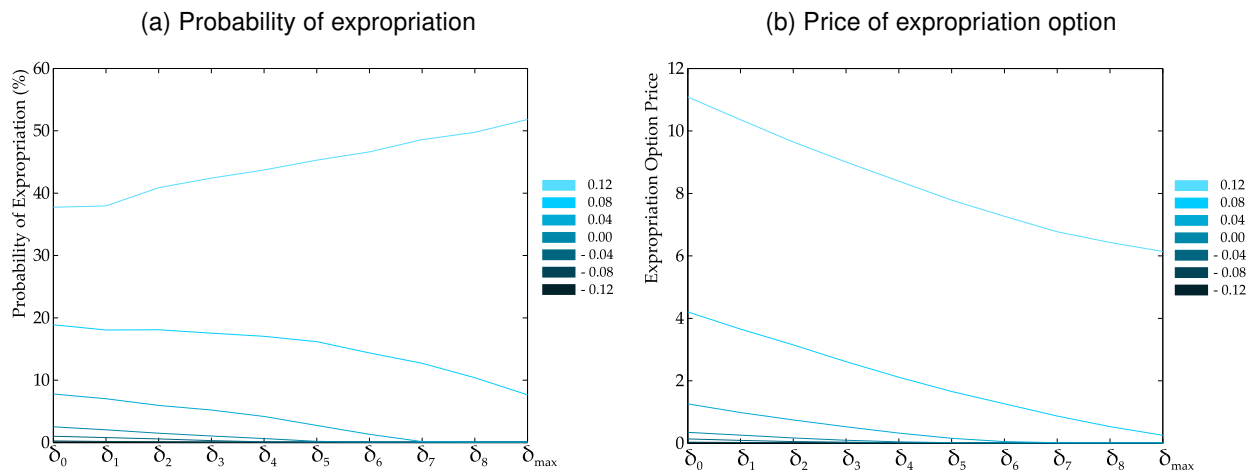
Figure 8: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by δ_i) in a nonlinear tax regime with different initial values for the spot price of the commodity S_0 , which varies according to the colour scale at the right of each figure (Case **C** in table 2).



more expropriation for more progressive taxes is less pronounced than it was in the oil price model. However, given an initial spot price that is large enough we will encounter an increased expropriation risk for a more progressive taxation regime, confirming the results shown previously for the variation of S_0 in the oil price model.

When we varied **the shape of the initial futures curve** for the case of the copper price model (case **D** in table 2), changing the parameter $\hat{v}_0$ from -0.12 to 0.12 (see figure 5b on page 19), we obtained the results shown in figure 9.

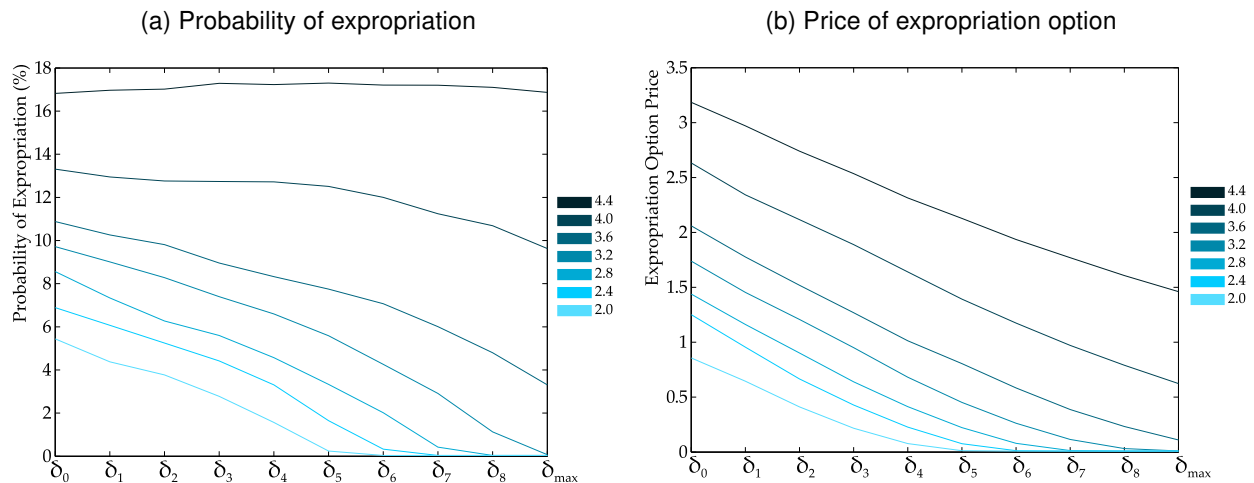
Figure 9: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by δ_i) in a nonlinear tax regime with different initial shapes of the futures curve obtained by varying the parameter $\hat{v}_0$, which varies according to the colour scale at the right of each figure (Case **D** in table 2).



Again, similarly to what happened when x_0 changed in the oil price model, if the initial futures contract curve that is in stronger contango, implying a higher value for $\hat{v}_0$, the risk of expropriation might not decrease with more tax progressiveness due to the anticipation of higher prices to come in the future.

The last case to be analysed for nonlinear equivalent tax regimes (case **E** in table 2) was the one **where there is a consensus over the long term price**. This was achieved by modifying the initial spot prices, in this case from $S_0 = 2.0$ to $S_0 = 4.4 \text{ USD/lb}$, while leaving constant the long term estimate at $S_T = 2.59 \text{ USD/lb}$, according to the estimations of the Chilean Budgetary Commission. We were interested in knowing if the long term average consensus affected the prospects of expropriation, and in this case that effectively did happen, as we can see in figure 10, where the risk of expropriation is substantially lower for even for the highest initial spot prices analysed as the long term expected price is considerably reduced.

Figure 10: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by δ_i) in a nonlinear tax regime and varying initial spot prices with long term average $S_T = 2.59 \text{ USD/lb}$, which vary according to the colour scale at the right of each figure (please note the inverted colour scale). This is case **E** in table 2.



Comments on the results under nonlinear taxes

After analysing the obtained results under equivalent nonlinear taxes, it is possible to make the following remarks:

Initial Spot Price For lower initial spot prices, a more progressive taxation regime effectively decreases the expropriation risk; however this effect disappears after a certain threshold has been reached. Then, for this critical initial spot price S_0^* and for every $S_0 \geq S_0^*$ a higher value of δ (i.e. a more progressive taxation regime) has the opposite effect, increasing the risk of expropriation up until reaching nearly 100% in the most extreme cases.

Futures Contracts Curves Shape When the initial futures contracts curve at time t_0 is in strong contango we observed the same effect that was noted in the case of large initial spot prices, that is, the unbiased expectation of future prices affects the expropriation risk, increasing it in the case of more progressive taxation.

Competing Dynamic Price Models The models showed a different response to higher prices, or futures contracts curves in strong contango: The oil price model showed a stronger increase in political risk with the increase of initial spot prices or the increase of futures contracts prices; however this same effect was shown—though muted—for the copper price model. The causes for this difference reside in the stronger reversion to the mean shown in simulated copper prices together with the differences in the volatility for both models, which allows the existence of a larger initial dispersion of prices in the oil price model.

Now we are going to revise how political risk varied with the introduction of equivalent windfall taxes, so we can contrast the results to the nonlinear tax regime.

3.3 Intuition and results: Windfall taxes

One can view the windfall tax as the sum of two flat corporate profit taxes. The first one is exacted at the rate τ_i over corporate profits²² while the second tax has a rate of $(1 - \tau_i)$ levelled over $[\max(S - S_{wf}, 0)]$, where S_{wf} is the windfall price and S the spot price at that moment. The resulting tax capture therefore will have a kink, as can be seen in figure 3b on page 12. Making that that the nature of the tax will change abruptly if prices are above the previously defined windfall price.

As in the case of the nonlinear tax, the ‘intuition’ is that a more progressive tax (i.e. both a lower initial tax rate τ_i and a lower windfall price S_{wf}) will increase the tax capture when prices are higher and therefore minimise political risk.

Oil price model under windfall taxes

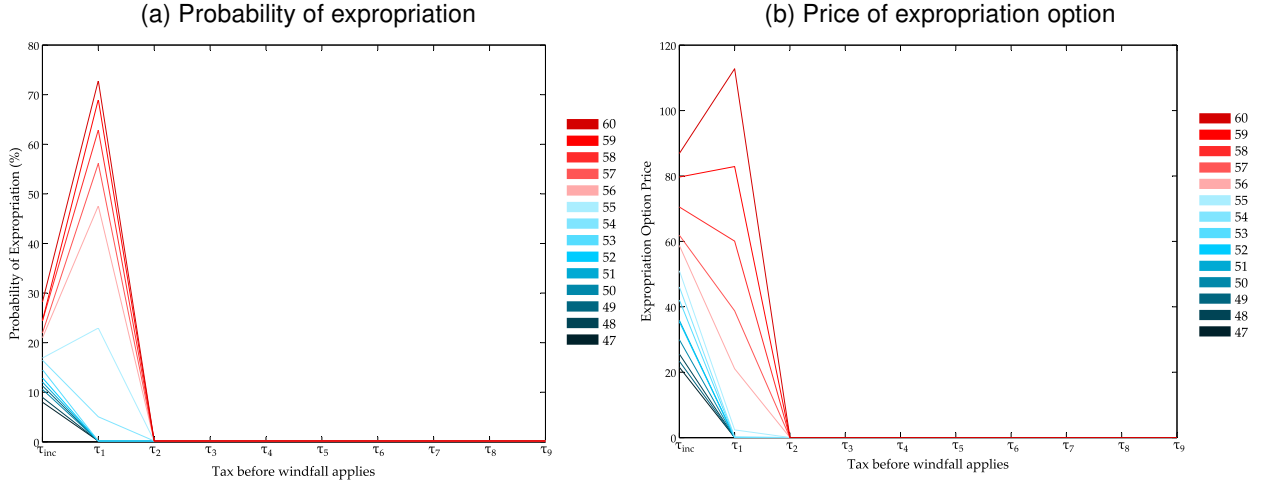
We started by replicating the methodology employed for the nonlinear tax regime in order to analyse the different results that were achieved when using a windfall tax regime. For this reason we started considering **how political risk varied with different initial spot prices** by using the same initial conditions as in the nonlinear tax regime (initial spot price, S_0 , between 47 and 60).

Similarly to what has been done in previous cases, the progressiveness in the taxation regime was modified by choosing a parameter τ_i , the tax rate for prices below the windfall price, setting the initial value equal to τ_{inc} , the baseline corporate income tax, and gradually reducing it until it reaches zero. As we can see on the definition for the windfall tax regime on page 10, this reduction in the initial tax rate will imply a decrease in the value of the windfall price, so that the lowest windfall price attainable in this experimental setting (while keeping the equivalence of the tax regime) would be $S_{wf}(\tau_9 = 0)$.

²²These are corporate profits to which the royalty rate τ_{roy} has already been deducted.

Therefore, the results obtained in this case—which corresponds to Case **F** in in table 2—can be seen in figure 11 below.

Figure 11: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of oil exploitation with increased progressiveness (given by τ_i) in a windfall tax regime with different initial spot prices S_0 , which vary according to the colour scale at the right of each figure (Case **F** in table 2).



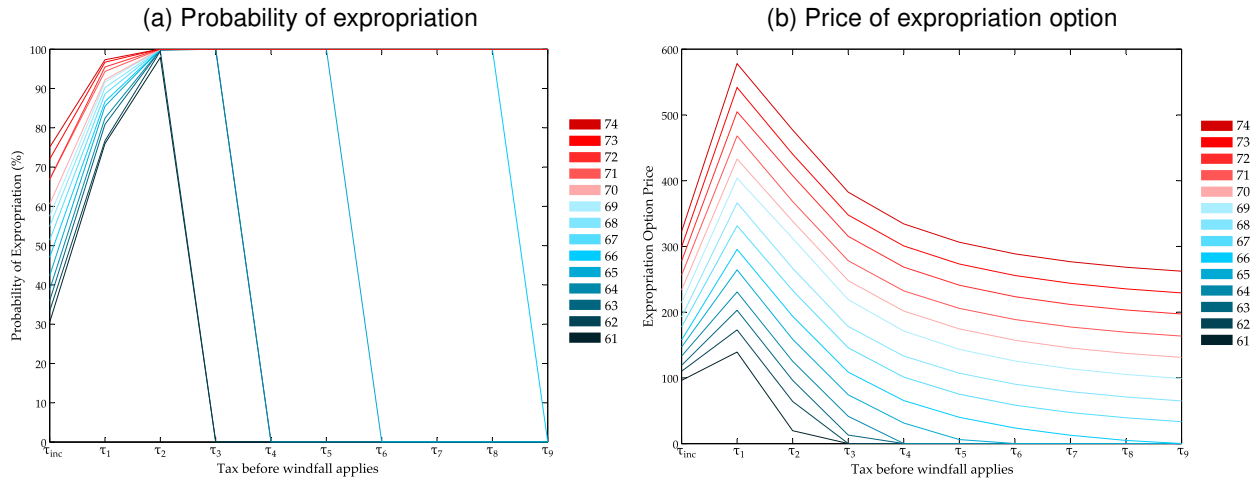
At first glance, we can note immediately two things: First, for an initial spot prices that are higher than a certain threshold (over 55 USD/bbl in this case) risk increases dramatically with the introduction of a windfall tax; however we can also note that this same expropriation risk goes to zero if taxes become more progressive. This result clearly contrasts with the nonlinear tax regime, where for the upper bound of prices in this interval we had an almost sure risk of expropriation.

This behaviour could be due to the windfall tax regime being more robust when taxes are more progressive for every single initial spot price or just for the studied interval. In order to clarify this we considered **an interval where initial spot prices were higher** (Case **G** in table 2) to see if the risk evolved in a different way with progressiveness as prices got higher. The results, for this new interval, with initial spot prices ranging from $S_0 = 61$ to $S_0 = 74$ are shown in figure12 below.

Interestingly, as prices increase, the level of progressiveness required to diminish political risk also increases. That is, if initial conditions are more favourable for expropriation, a higher degree of progressiveness is going to be required, until the price is so high ($S_0 = 67$ USD/bbl or higher in this case) that even with maximum progressiveness an equivalent tax regime is going to be unable to eliminate the political risk. Moreover, we can also see how after the price of the option jumps due to an increase in the expropriation probability it then slowly starts to decrease as this expropriation is taking place in worse conditions as tax progressiveness increases.

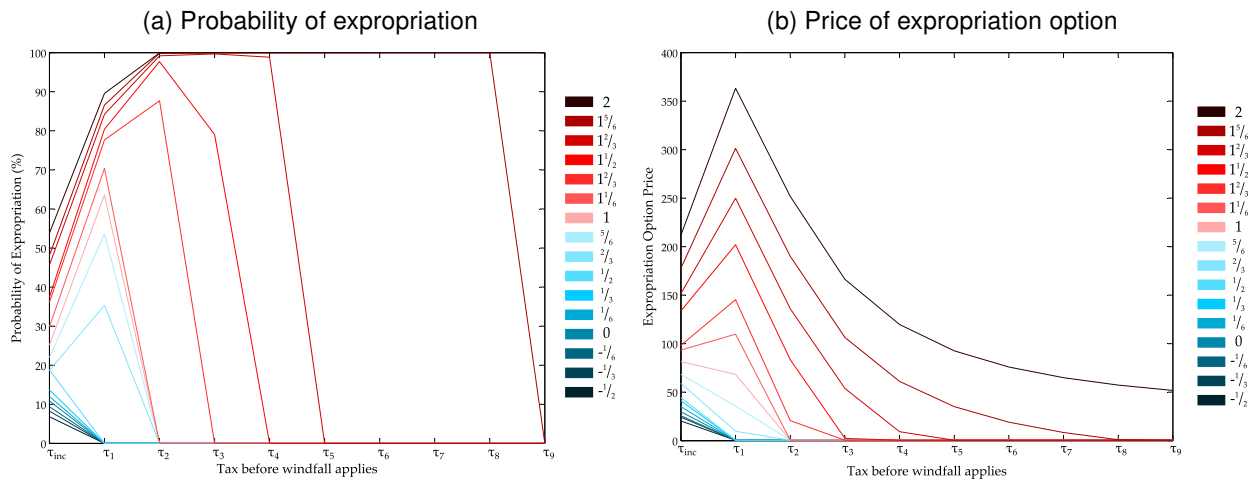
Similarly, to check whether **the shape of futures contracts curve generated similar results** when this curve was in strong contango, we varied the parameter x_0 (Case **H** in table 2) from

Figure 12: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of oil exploitation with increased progressiveness (given by τ_i) in a windfall tax regime with different initial spot prices S_0 , which vary according to the colour scale at the right of each figure (Case **G** in table 2).



–0.5 to 2.0 keeping the initial spot price constant at $S_0 = 50 \text{ USD/bbl}$. The results can be seen in figure 13 below.

Figure 13: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of oil exploitation with increased progressiveness (given by τ_i) in a windfall tax regime with different shapes of the futures curve, obtained by varying parameter x_0 , which changes according to the colour scale at the right of each figure (Case **H** in table 2).



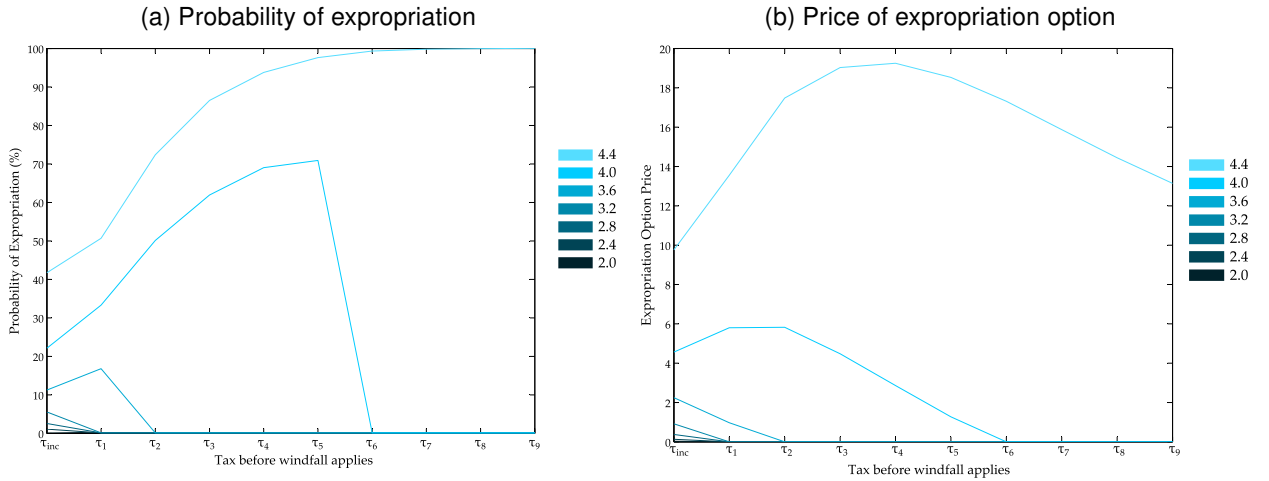
The results thus obtained are similar to the results of the extended initial spot price range (case **G**) for futures curve in contango even though the windfall tax regime proved to be quite robust at this initial spot price.

We will now look at the response that the copper price model has to the windfall price as it was done previously with the nonlinear tax regime.

Copper price model under windfall taxes

We tested different sets of copper prices generated with the Cortázar and Schwartz (2003) model using varied initial conditions to see the response they had to the real options expropriation model under windfall taxes. In the first case—Case I in table 2—we replicated **the change in the initial spot prices**, S_0 , from 2.0 to 4.4 USD/lb , as it was done in the nonlinear tax regime so that we can observe the result in figure 14 below.

Figure 14: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by τ_i) in a windfall tax regime with different initial values for the spot price of the commodity S_0 , which varies according to the colour scale at the right of each figure (Case I in table 2).



Here we can note that even if there are noticeable differences from the crude oil case, a common aspect is that for a given interval in prices the expropriation risk increases with progressiveness only to fall abruptly; and if prices are high enough (4.4 USD/lb in this setting), no equivalent tax regime can decrease the risk of expropriation risk.

When the **shape of the futures price curve was changed**, also in similarity with the nonlinear case going from backwardation (where the initial price model parameter $\hat{v}_0 = -0.12$) to a strong contango (with the same parameter $\hat{v}_0 = 0.12$), we obtain results for the expropriation model that can be seen in figure 15 below.

These results are not dissimilar to those that can be obtained from **changing the initial spot price, S_0 while keeping the long term copper price constant at $S_T = 2.59$ USD/lb** , as can be seen in figure 16b below.

In both cases what we see is a repetition of the pattern from case I above, with the caveat that in case J—the variation of the futures contracts curves—this curve is not in a strong enough contango to make expropriation inevitable when progressiveness is maximal. The same can be said from keeping the long term outlook price constant, the range that was sufficient to make expropriation unavoidable in case I does not suffice due to the corresponding futures

Figure 15: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by τ_i) in a windfall tax regime and different initial shape of the futures curve by varying the parameter $\hat{v}_0$, which varies according to the colour scale at the right of each figure (Case J in table 2).

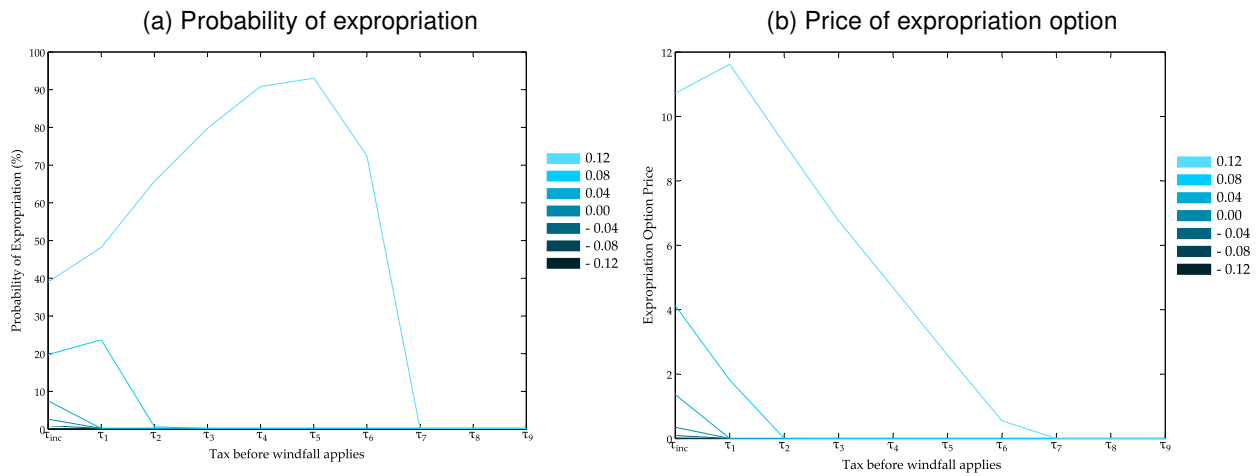
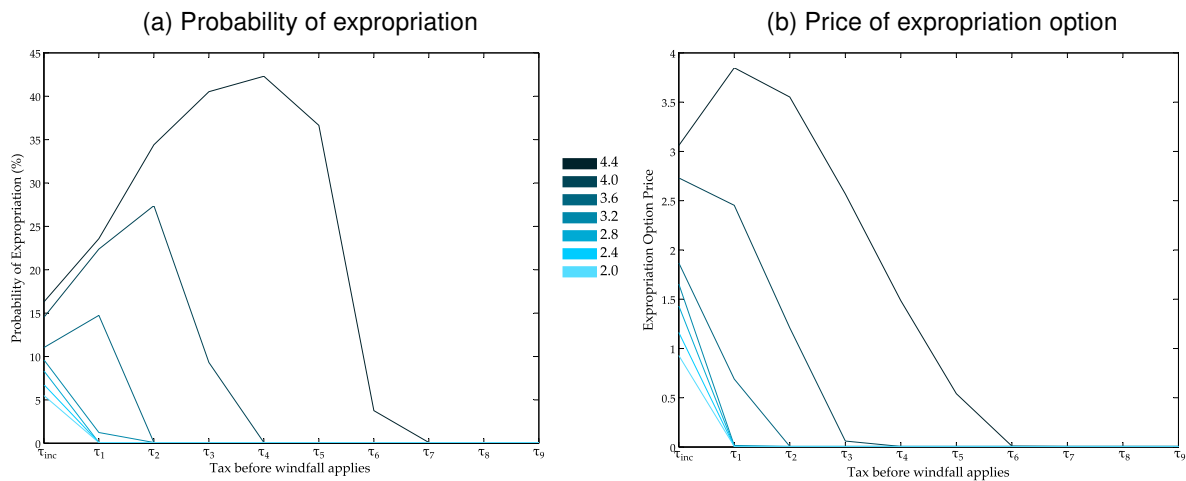


Figure 16: Evolution of expropriation parameters (total probability and price of the expropriation option) in the case of copper mining with increased progressiveness (given by τ_i) in a windfall tax regime and varying initial spot prices with long term average $S_T = 2.59 USD/lb$, which vary according to the colour scale at the right of each figure (please note the inverted colour scale). This is case K in table 2.



curves being in backwardation.²³

Comments on the results under windfall taxes

The results differ somewhat to those obtained with nonlinear taxes, of these differences we can mention the following:

Initial Spot Price We have already seen that, all else being equal, higher initial spot prices

²³As can be seen in figure 5c on page 19.

increased the expropriation risk for the project. However with windfall taxes we saw that there are not one, but two different initial threshold prices. If the initial prices were low enough²⁴ then the introduction of a more progressive taxation regime would decrease the political risk of the operations. However, for a range of initial spot prices higher than this threshold the introduction of a more progressive taxation regime is going to generate a jump in the riskiness of the operations, a risk that will eventually disappear as taxes become more progressive. Finally, if the initial spot prices are high enough, then no increase in the progressiveness of the equivalent windfall tax regime can make the risk to decrease.²⁵ Though there are clear differences between the crude oil and the copper price model, the phenomenon of two threshold prices can also be seen for copper in figure 14

Shape of the Futures Contracts Curve or Constant Expectation of Long Term Price The results in these cases were quite similar to those observed when the initial spot price was the one that changed. However for the copper price model,²⁶ the range of the curve for futures contracts did not go into a sufficiently strong contango, nor the initial spot prices were sufficiently high to reach a second threshold, though it can be argued that if these initial conditions were sufficiently adverse, the pattern of increased risk would repeat itself again.

Competing price models It is clear that for every type of initial scenario the copper price model was more robust when progressiveness in taxation was introduced.

Below we will try to explain why the introduction of progressiveness in taxation might increase the risk for natural resource projects as well as the different effect that this same progressiveness has on different taxation regimes.

3.4 Impatience and expropriation risk: An Analysis of the results

So far we have been able to see that equivalent and more progressive tax regimes can have counterintuitive effects on the political risk of operations if they are adopted under adverse initial conditions. In this section we will analyse the reasons for the departure from this intuition, in such a way that it provides some insight for a more appropriate behaviour of both the industry and the governments in such cases.

A brief summary of the results

The purpose of this paper is to see how the introduction of an equivalent taxation regime can affect the political risk of a natural resources project; we can summarise the effects that these different tax regimes had on the analysed price models in table 3 below. There, robustness to the introduction of a more progressive taxation regime is characterised according to two dimensions: *Fiscal Stability*, or the decrease in risk that is consequential to the introduction

²⁴The lowest initial spot prices of case **F** as can be seen in figure 11.

²⁵This can be observed for the highest initial spot prices of case **G** as can be seen in figure 12.

²⁶Cases J and K in table 2 and figures 15 and 16 respectively.

of a new progressive tax regime; and *Smoothness*, or how this risk can change with minor variations of the progressiveness parameter.

Table 3: A summary of the robustness of introducing a progressive tax regime for different price models according to two dimensions: ‘Fiscal Stability’ and ‘Smoothness’

		Price Model	
		Crude Oil	Copper
Nonlinear		↓↓ Fiscal Stability ↑ Smoothness	↓ Fiscal Stability ↑↑ Smoothness
Windfall		↑ Fiscal Stability ↓↓ Smoothness	↑↑ Fiscal Stability ↓ Smoothness

As can be seen, windfall taxes are more robust in having a lower risk than what could be expected from the same initial conditions than a nonlinear tax. But this comes at the cost of smoothness, where a minor variation in the progressiveness parameter can have a major change on the riskiness of the project. A windfall tax can thus be less risky with high progressiveness but more volatile under variations of the progressiveness parameter.

The timing of the expropriation decision

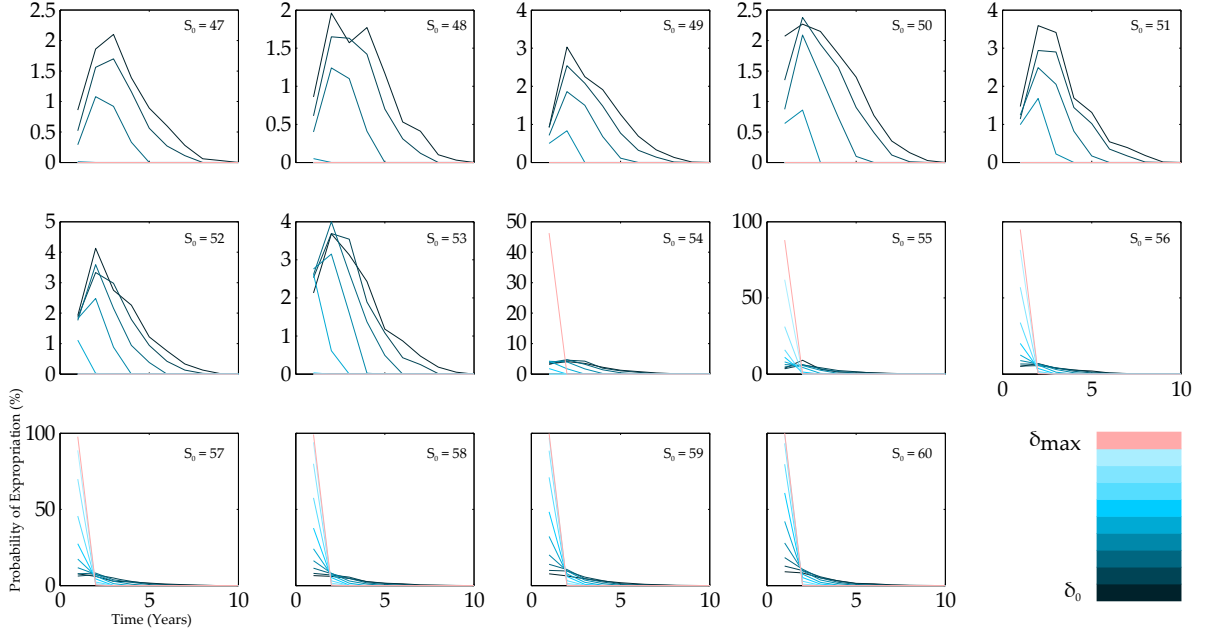
The counterintuitive result, that the risk of expropriation increases with progressiveness, has been thoroughly revised in the past section. Nevertheless, when we calculate the total probability of expropriation²⁷ every single expropriation decision is treated as equal, while we know that there are qualitative differences if there is an early decision to expropriate.

If this happens at the earliest period, t_1 , the value of the operation for the private operating firm V_{firm}^{exp} will be reduced to the compensation payment over the remaining lifetime of the mine or oilfield, $K_1 \cdot T$, while if the expropriation decision is carried out later, then the private firm is going to participate in some of the upside or downside of the market. It is therefore interesting to observe the evolution of the timing of expropriation probability with increased progressiveness: The qualitative response to expropriation timing when facing increased progressiveness in taxation was almost identical for both the nonlinear and windfall tax regimes. We can see this pattern in figure 17 below, where we show the timing of expropriation of one of the analysed cases (Case **A** from table 2), in which there was a variation of the initial spot price, S_0 , for nonlinear taxes.

Here we can see that for low initial spot prices, S_0 from 47 to 53 *USD/bbl* in this case, the expropriation decision peaks at either the second or third year of operation, with expropriation risks being lower for more progressiveness—i.e. higher values of δ (lighter blue and red lines)—implying that the ‘intuition’ holds; however we should note that almost all expropriation in those cases with higher progressiveness takes place in the first two years.

²⁷Using equation (23) on page 15.

Figure 17: Timing of expropriation probability for the oil production case using the Trolle and Schwartz (2009) model under a nonlinear tax regime and different initial spot prices S_0 , with progressiveness parameter δ between δ_0 and δ_{max} , each line representing a different value of δ according to the colour scale in the bottom right corner.



At higher initial spot prices the picture reverses and more progressive taxation induces a higher probability of expropriation (the counterintuitive result). Moreover, results are qualitatively different in that almost all expropriation takes place in the first year,²⁸ The fact that almost all expropriation decisions take place as early as possible when the tax regime is more progressive contrasts with the baseline case, and indicates that there is some intrinsic difference in the dynamics of the expropriation process when spot prices are high; something to be explored below.

Payoff distribution and impatience

As has been discussed in previous sections, the expropriation model relies on an American-style option to value the expropriation risk; an option with a payoff function $\Pi(i, t_j, k)$ for each path i at time t_j with progressiveness parameter k , which has the following form:

$$\Pi(i, t_j, k) = \max(V_{gain}(i, t_j) - V_{cost}(i, t_j, k), 0). \quad (16)$$

Where $V_{gain}(i, t_j)$ and $V_{cost}(i, t_j, k)$ represent the discounted gain and loss of exercising the option at each period t_j , so that if a government were completely short-sighted it would find attractive to expropriate whenever $\Pi(i, t_j, k) > 0$. This behaviour is prevented given that there is an alternative to exercise an option immediately, and that is to wait until the option itself becomes more *in the money*; if there are substantial prospects of further valuation we would

²⁸If we further disaggregate the data we can see that in the cases where the initial spot price is high enough, when progressiveness increases *all expropriation* takes place in the *first period of the first year*.

see that governments would restrain from their willingness to expropriate in the short term even if their payoff function is positive.

As a consequence, there are going to be occasions where even if the payoff is positive, the government is going to refrain from expropriating a mine/oil field given that there are sufficient prospects of further valuation of the option; we can say that in those cases a government is not being *impatient*. In order to see how this patience, or the lack of it, evolves with a more progressive tax regime we can define the *Impatience Coefficient*, as the quotient between the amount of expropriations that take place and the total amount of price paths that have a payoff function that is positive, the equation being the following:

$$Impatience\ Coeff(k) = \frac{\sum_{i=1}^M \mathbf{I}_{[t_N > t_{Z_i}]}}{\sum_{i=1}^M \max(\mathbf{I}_{[\Pi(i, t_j, k) > 0]}, \forall t_j \in \{t_1, \dots, t_{N-1}\})} . \quad (24)$$

Where M is the total number of simulated price paths, $\mathbf{I}_{[t_N > t_{Z_i}]}$ is an indicator function equal to one if an expropriation decision is taken in price path i and the denominator is equivalent to the total number of price paths that have at least one payoff function that is positive through the lifespan of the mine/oil field.

The *Impatience Coefficient* is therefore going to be a value between 0 and 1, where 1 implies complete impatience: In that case, the government will expropriate any path that has a positive payoff function at least at one period. We can see how these coefficients evolve with progressiveness for every price model and tax regime when changing initial spot prices in figure 18 below.

The Impatience Coefficient increases with progressiveness, **even in those cases where the overall expropriation risk is decreasing**, what happens is that there are fewer prospects of an increase in the payoff function in the future. These results are also replicated with other initial conditions, such as the change in the initial futures contracts curves.

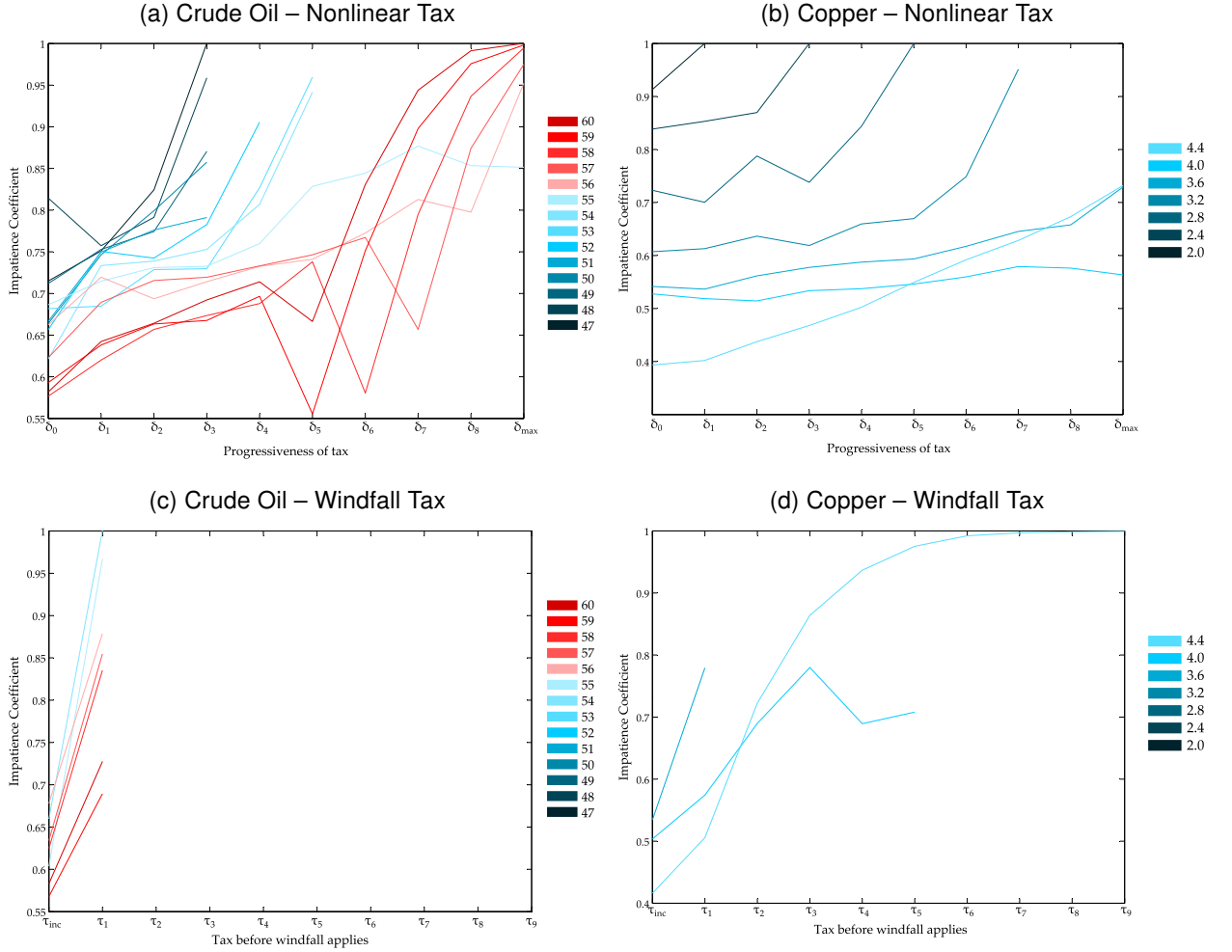
This variation in impatience can be better understood if we see how the values of the payoff function evolve with increased progressiveness. For example, in the case of the crude oil price model with nonlinear taxes (case **A** from table 2) the evolution of the expropriation probability and of the value of the expropriation option for two initial conditions under varying progressiveness can be seen in table 4 below.

We can now see how the distribution of the positive values from the payoff function, $\Pi(i, t_j, \delta_i)$ evolved under these initial conditions; these values are presented in figures 19 and 20.

From table 4 we know that when $S_0 = 50\ USD/bbl$ an increase in the progressiveness parameter δ will rapidly decrease the probability of expropriation from almost 11% to zero: figure 19 shows us that the reason for this decline, that the total number of price paths that had a positive payoff decreased with the increase in δ_i until finally disappearing when the parameter was δ_4 or larger.

In the case where $S_0 = 60$, which can be seen in figure 20, an increase in the progressiveness parameter δ also decreases the magnitude of the most extreme positive values for the payoff function. However we can also see that the distribution of these payoffs starts skewing towards the initial periods, and that there is a higher frequency of positive payoffs at the beginning. It is this concentration of positive payoffs at the initial periods of exploitation and the corresponding skewness of the payoff function which suppresses the probabilities of appreciation for the American-style option what prompts the option to be exercised.

Figure 18: Evolution of the *Impatience Coefficient* for the Crude Oil and Copper price models under Nonlinear and Windfall tax regimes with increased progressiveness in the applicable tax. Initial spot price of the commodities varies according to the colour scale at the right of each figure. These cases correspond to cases **A**, **C**, **F**, and **I** in table 2 respectively.



We can extend this same analysis to the windfall tax, where the same phenomenon applies—reduction in the value of the payoff function, increase in its skewness, and an initial payoff value that it is positive—the main difference being that in some range the skewness is rapidly introduced (and hence, a jump in the expropriation risk), but as the windfall tax, τ_{wf} , becomes more progressive the magnitude of the payoff function decreases and so there is no payoff function with positive value; this can be seen more clearly in figure 21, which shows us the payoff function for an initial price of oil of $S_0 = 60$ under windfall taxes.

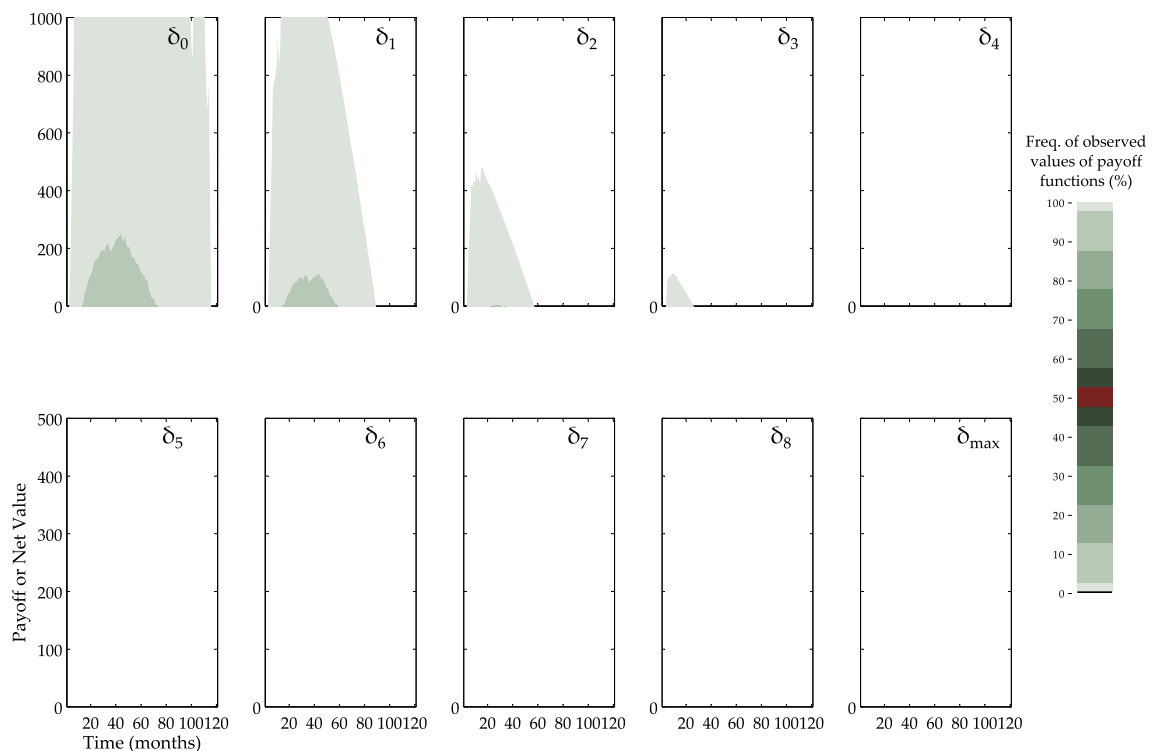
In this case, as we could see previously in figure 11a, expropriation jumps when the windfall tax regime is introduced ($\tau_{wf} = \tau_1$) and then plummets. What is interesting in this case is the surge in expropriation risk that is caused by the extreme skewness of the payoff function; as the windfall acts like a barrier to further appreciation of tax collection for prices over S_{wf1} .

In summary, when taxes become too progressive and initial conditions are not favourable (high initial prices or futures curves into strong contango), the maximum magnitude of the payoff

Table 4: Expropriation probability and value of the government's expropriation option for initial spot prices $S_0 = 50$ and $S_0 = 60$ in the crude oil price model under varying progressiveness from the nonlinear taxation regime.

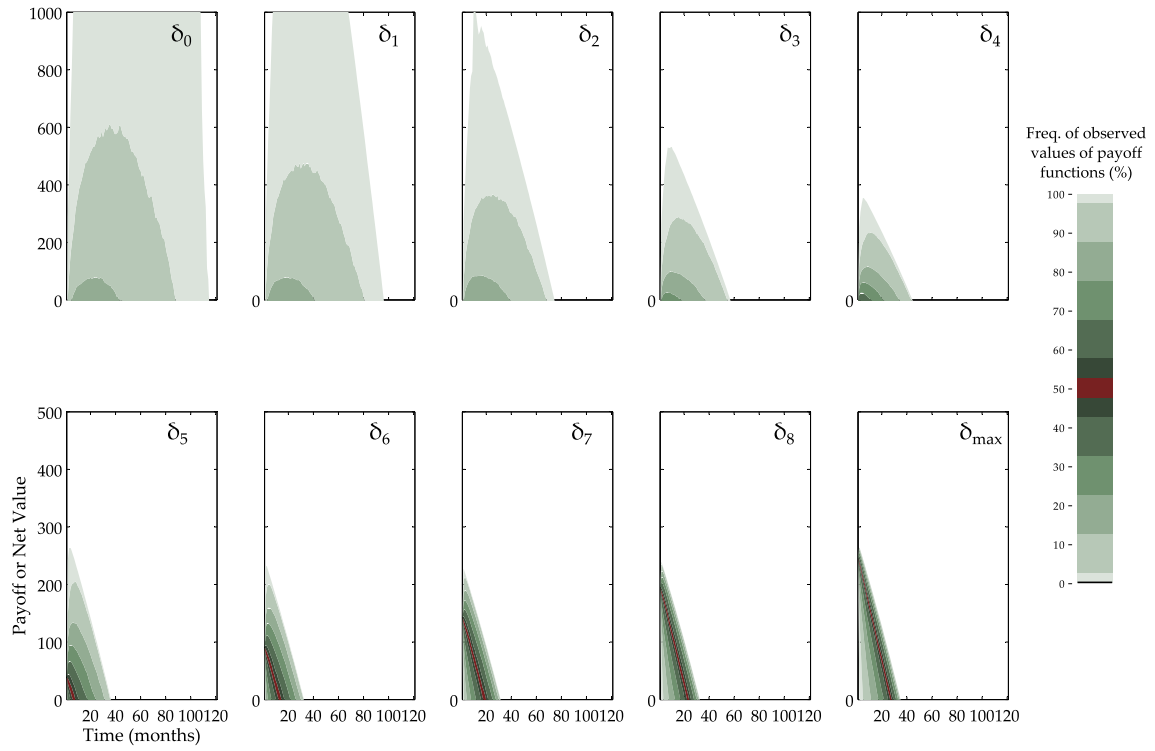
	Expropriation Prob		Price of Exp. Option	
	S_0		S_0	
	50	60	50	60
δ_0	10.98%	28.53%	34.16	88.35
δ_1	8.75%	33.41%	19.11	71.45
δ_2	5.22%	37.23%	7.67	59.82
δ_3	1.50%	42.60%	1.38	53.30
δ_4	0.00%	51.96%	0.00	55.09
δ_5	0.00%	66.82%	0.00	67.33
δ_6	0.00%	82.66%	0.00	92.72
δ_7	0.00%	94.37%	0.00	131.14
δ_8	0.00%	99.11%	0.00	178.51
δ_{max}	0.00%	99.98%	0.00	228.11

Figure 19: Evolution of the positive values of the undiscounted payoff function $\Pi(i, t_j, \delta)$ with increasing progressiveness parameter δ (nonlinear tax regime) when $S_0 = 50$. Colours represent the distribution of the different price paths when $M = 10,000$.



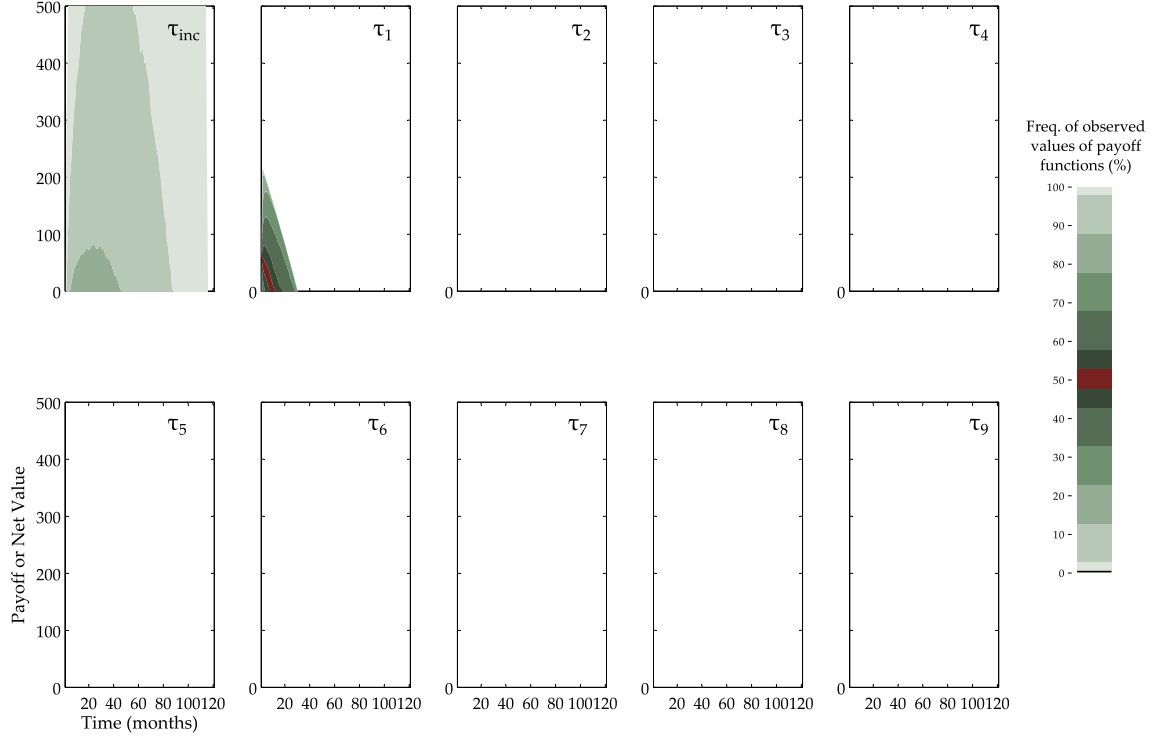
function decreases, but at the same time the initial conditions of this payoff begin to be *in the money*, the option has an immediate value. And it is this combination of lower maximum values for payoff that decreases expectations and initial favourable conditions that make expropriation

Figure 20: Evolution of the positive values of the undiscounted payoff function $\Pi(i, t_j, \delta)$ with increasing progressiveness parameter δ (nonlinear tax regime) when $S_0 = 60$. Colours represent the distribution of the different price paths when $M = 10,000$.



favourable under more progressive tax regimes which make equivalent progressive taxation to be riskier when initial conditions exceed certain thresholds.

Figure 21: Evolution of the positive values of the undiscounted payoff function $\Pi(i, t_j, \tau_{wf})$ with increasing progressiveness parameter τ_{wf} (windfall tax regime) when $S_0 = 60$. Colours represent the distribution of the different price paths when $M = 10,000$.



4 Conclusions

As we have been able to see in this paper, the initial hypothesis described as the ‘intuition’—that a more progressive taxation regime lowers the political risk of natural resource operations—is only partially true. Whenever current spot prices or the unbiased expectation of future prices are sufficiently large, the government will become more impatient after introducing an equivalent taxation regime that is more progressive, expropriating more and in worse conditions than if the baseline corporate profit tax were maintained.

That being said, it must be still emphasised that for a broad range of prices, a more progressive tax regime effectively decreases the probability of expropriation and enhances the total social value of the mining/oil operations.

Some conclusions that apply to every model analysed in this paper are the following.

Long term expectations matter

The expropriation model as set by Schwartz and Trolle (2010), implicitly relies on the assumption that the no-arbitrage condition holds at every moment. As we considered that both firms and the government were risk neutral, this condition allows us to replace the value of the initially expected spot price of the underlying commodity at time t_j , $E_{t_0}[S_{t_j}]$ with the value at t_0 of the futures contract maturing at t_j , $F(t_0, t_j)$.

The consequence of this is that the shape of the futures curve at the initial moment has a

long term effect, affecting the firm's and the government's expectations of income from the operations up until the end of the life of the mine/oil field. This is due to the dynamics of the price models being used, a dynamic behaviour that is further enforced by the 'calibration' of price paths as detailed on page 47.

This long term dependence of the price distribution implies that the risks of expropriation can be foreseen ex ante by both parties, and therefore that firms will value their expected cash flows at an additional discount when expropriation risks seem to be higher.

Negotiating change

Another conclusion that can be drawn from the results is that negotiating a change in the tax regime becomes especially sensitive when prices or the expectation of future prices are high. To minimise compensation costs, governments can be tempted to negotiate a tax modification that leaves companies' expected income after taxes unchanged, and so could be willingly agreed by both parties. However, as we have seen in this paper, changing the tax regime under unfavourable initial conditions for an equivalent but more progressive taxation structure will increase the political risks of the operations and decrease the total social value for firms and governments alike.

Therefore, negotiations to change a tax regime should be done considering long term price expectations, and in those cases where the futures prices are high, the government and the firms should abandon the idea that a tax reform that does not affect the expected income for the companies can be carried out without major side effects. The answer in this case is to choose a different sort of tax reform, one that increases reputational costs, K_2 , or that compensates the increased tax burden on companies with upfront payments or tax breaks that force governments to become more patient if they wish to retrieve taxes from accumulated gains.

The role of nonlinear and windfall tax regimes

As we mentioned at the beginning of this paper, the definition of an equivalent taxation regime is sufficiently broad to allow a number of possible taxation regimes that vary in its progressiveness. This paper focused on only two families of taxes, a nonlinear tax regime, following the suggestion of Stroebe and van Benthem (2010), and a windfall tax regime which is inspired by the optimal tax proposed by Engel and Fischer (2010). As could be noted in the results, the windfall tax regime appeared to offer more assurances of fiscal stability when initial conditions were equivalent and taxes were highly progressive.

However, this increased robustness in fiscal stability terms must be weighted against the increased volatility with respect to the progressiveness parameter shown by the windfall tax regime. A small variation in τ_{wf} can have a huge incidence in perceived risks, and therefore in those cases where there is a higher uncertainty about the value of the parameters, the inclusion of a nonlinear tax regime that decreases the risk would be preferable as it would result in a tax regime that withstand fluctuations of the parameters without a massive jump in risk.

Extending the model

All the variations of the model presented in this paper take production parameters as certain. However, as we have just seen, the inclusion of some volatility in the magnitudes can have a major effect in the perceived political risk; and more so if the equivalent taxation regime that's being implemented is highly sensitive to these changes.

Even when there are national resource companies in operation and the extraction process is well known, some uncertainty about the costs of operation will remain²⁹ and therefore it would be interesting to see how uncertainty on C_{firm} and C_{gov} would affect the decision process.

Moreover, the size of the mine/oil field is known only approximately when the operations begin; and the exploitation rate is going to be driven by this process as well as by the current and expected price of the commodity. A variation of the multiple real options analysis by Aleksandrov and Espinoza (2009) may represent the ability of natural resources companies to adapt their production rate according to market and geological news.

The uncertainty of production variables can be strategically managed in order to minimise risks; therefore providing an effective complement to fiscal stability promises made without commitment. Considering how this can be done will provide additional tools for governments so that they can maximise the social benefit that arises from the exploitation of their national resources.

²⁹See for example COCHILCO (2008) for an estimation of copper extraction costs in Chile.

References

- Aleksandrov, N. & Espinoza, R. (2009). *Optimal oil extraction as a multiple real option* (Working Paper).
- Boadway, R. & Keen, M. (2010). Theoretical perspectives on resource tax design. In P. Daniel, M. Keen & C. McPherson (Eds.), *The taxation of petroleum and minerals: Principles, problems and practice* (p. 13-74). Routledge.
- Bohn, H. & Deacon, R. T. (2000, June). Ownership risk, investment, and the use of natural resources. *The American Economic Review*, 90(3), pp. 526-549.
- Brandimarte, P. (2006). *Numerical methods in finance and economics: a matlab-based introduction* (2nd ed.). Wiley-Interscience.
- COCHILCO. (2008, December). *Costos de la minería: ¿Cuánto impactan los insumos en la industria del cobre?* (Documento de Estudio No. 17/08). Comisión Chilena del Cobre.
- Congreso Nacional de Chile. (2005, June). *Historia de la ley número 20.026: Establece un impuesto específico a la actividad minera* (Law Proceedings). Biblioteca del Congreso Nacional de Chile.
- Cortázar, G. & Schwartz, E. S. (1998). Monte Carlo evaluation model of an undeveloped oil field. *Journal of Energy Finance & Development*, 3(1), pp. 73-84.
- Cortázar, G. & Schwartz, E. S. (2003, May). Implementing a stochastic model for oil futures prices. *Energy Economics*, 25(3), pp. 215-238.
- DIPRES. (2010, August). Acta de resultados del comité consultivo del precio de referencia del cobre. In DIPRES, Chile (Ed.), . DIPRES, Dirección de Presupuestos de Chile.
- Engel, E. & Fischer, R. D. (2010, June). Optimal resource extraction contracts under threat of expropriation. In W. Hogan & F. Sturzenegger (Eds.), *The natural resources trap* (p. 161-195). The MIT Press.
- Garnaut, R. & Ross, A. C. (1975, June). Uncertainty, risk aversion and the taxing of natural resource projects. *The Economic Journal*, 85(338), pp. 272-287.
- Hotelling, H. (1931, April). The economics of exhaustible resources. *The Journal of Political Economy*, 39(2), pp. 137-175.
- Johnston, D. (2008). Changing fiscal landscape. *Journal of World Energy Law & Business*, 1(1), pp. 31-54.
- Longstaff, F. A. & Schwartz, E. S. (2001). Valuing american options by simulation: A simple least-squares approach. *Review of Financial Studies*, 14(1), pp. 113-147.
- Mahajan, A. (1990). Pricing expropriation risk. *Financial Management*, 19(4), pp. 77-86.
- Osmundsen, P. (1998, November). Dynamic taxation of non-renewable natural resources under asymmetric information about reserves. *The Canadian Journal of Economics*, 31(4), pp. 933-951.
- Osmundsen, P. (2010). Time consistency in petroleum taxation: Lessons from Norway. In P. Daniel, M. Keen & C. McPherson (Eds.), *The taxation of petroleum and minerals: Principles, problems and practice* (p. 425-444). Routledge.
- Otto, J. (2000, November). *Mining taxation in developing countries* (Study prepared for UNCTAD). United Nations Conference on Trade and Development.
- Rigobón, R. (2010, June). Dealing with expropriations: General guidelines for oil production contracts. In W. Hogan & F. Sturzenegger (Eds.), *The natural resources trap* (p. 227-254). The MIT Press.

- Schwartz, E. & Trolle, A. B. (2010, June). Pricing expropriation risk in natural resource contracts. In W. Hogan & F. Sturzenegger (Eds.), *The natural resources trap* (p. 263-287). The MIT Press.
- Stroebel, J. & van Benthem, A. (2010, September). *Resource extraction contracts under threat of expropriation: Theory and evidence* (Working Paper). Stanford University.
- Titelman Goren, E. (2007). Presentación ante la comisión investigadora sobre CODELCO-Chile. In Comisión Chilena del Cobre COCHILCO (Ed.), . Report presented before the Chilean Parliament's Investigative Committee, 10 December 2007.
- Trolle, A. B. & Schwartz, E. S. (2009). Unspanned stochastic volatility and the pricing of commodity derivatives. *Review of Financial Studies*, 22(11), pp. 4423-4461.

Appendix

A Price Paths for Monte Carlo Simulations

The dynamic price models used in this paper generate a sequence of spot prices S_{i,t_j} for each commodity, together with a corresponding futures contracts curve for every single price path available in the simulation at every period and for every maturity. These price paths and futures contracts are going to depend on some initial conditions (of which initial spot price and the shape of the futures contracts curve at the initial time are most important) and on the realised values of a random Wiener processes. Coincidentally, in these two models the stochastic element is described using three variables dz_1^* , dz_2^* and dz_3^* which are correlated Wiener processes so that:

$$\begin{aligned} dz_1^* dz_2^* &= \rho_{12} dt, \\ dz_1^* dz_3^* &= \rho_{13} dt, \\ dz_2^* dz_3^* &= \rho_{23} dt. \end{aligned} \quad (25)$$

Hence, if we know the correlation parameters, ρ_{ij} , we can generate these elements and the subsequent price paths from independent standardised normal random variables using a Cholesky decomposition.

A.1 The Trolle and Schwartz (2009) price model

In the model presented by Trolle and Schwartz (2009) the dynamics of the spot price of a commodity are based on a *single stochastic volatility model* that relies on Itô's lemma to replace the spot price for $s = \log(S)$. Therefore, for each generated path i we will have the following equations:

$$ds_{i,t} = \left(\varphi + \alpha x_{i,t} + \alpha \phi_{i,t} - \frac{1}{2} \sigma_S^2 v_{i,t} \right) dt + \sigma_S \sqrt{v_{i,t}} dz_{1,i,t}^*, \quad (26)$$

$$dx_{i,t} = \left(-\gamma x_{i,t} - \left(\frac{\alpha}{\gamma} + \rho_{12} \sigma_S \right) v_{i,t} \right) dt + \sqrt{v_{i,t}} dz_{2,i,t}^*, \quad (27)$$

$$dv_{i,t} = (1 - \kappa_v v_{i,t}) dt + \sqrt{v_{i,t}} dz_{3,i,t}^*, \quad (28)$$

$$d\phi_{i,t} = \left(\frac{\alpha}{\gamma} v_{i,t} - 2\gamma \phi_{i,t} \right) dt. \quad (29)$$

As can be noted, the stochastic volatility is described by equation (28), and affects the dynamics of both the log of the spot price, $s_{i,t}$, and of the futures curves parameter, $x_{i,t}$. Given certain initial conditions we can simulate paths for the spot price using equation (26); and for each path i , where $i \in \{1, \dots, M\}$, we can generate futures contracts at every time t_j maturing at time t_l according to:

$$F_i(t_j, t_l) = \exp \left[\varphi(t_l - t_j) + s_{i,t_j} + \frac{\alpha}{\gamma} (1 - e^{-\gamma(t_l - t_j)}) x_{i,t_j} + \frac{\alpha}{2\gamma} (1 - e^{-2\gamma(t_l - t_j)}) \phi_{i,t_j} \right]. \quad (30)$$

A.2 The Cortázar and Schwartz (2003) price model

The article by Cortázar and Schwartz (2003) introduces us to another mean-reversal model for generating prices that are consistent with a no-arbitrage condition in their futures curve. In this paper this model was used to generate a representation of the behaviour of copper prices and their corresponding futures contracts at every time t , by simulating the spot, S_t , the demeaned convenience yield, y_t , and the expected long-term spot price return v_t (with a long-term average $\bar{v}$ as a parameter). For the risk-neutral process we will have the following equations for each generated path i :

$$dS_{i,t} = (v_{i,t} - y_{i,t} - \lambda_1)S_{i,t}dt + \sigma_1 S_{i,t}dz_{1,i,t}^*, \quad (31)$$

$$dy_{i,t} = (-\kappa y_{i,t} - \lambda_2)dt + \sigma_2 dz_{2,i,t}^*, \quad (32)$$

$$dv_{i,t} = a(\bar{v} - v_{i,t} - \lambda_3)dt + \sigma_3 dz_{3,i,t}^*, \quad (33)$$

where, as before, $dz_{1,i,t}^*$, $dz_{2,i,t}^*$, and $dz_{3,i,t}^*$ are correlated Wiener processes. After some rearrangement of the variables we can minimise the number of parameters to be found and improve the fit of the process, therefore we consider the following new variables:

$$\hat{y}_{i,t} = y_{i,t} + \frac{\lambda_2}{\kappa}, \quad (34)$$

$$\hat{v}_{i,t} = v_{i,t} - \lambda_1 + \frac{\lambda_2}{\kappa}, \quad (35)$$

and

$$\beta = \frac{a\bar{\hat{v}} - \lambda_3 + \sigma_{13}}{a^2}; \quad (36)$$

where $\bar{\hat{v}}$ is the long term average of $\hat{v}$, and is calculated ex post. We thus have now the following simplified dynamic equations without the use of the risk adjustment factor λ_i :

$$dS_{i,t} = (\hat{v}_{i,t} - \hat{y}_{i,t})S_{i,t}dt + \sigma_1 S_{i,t}dz_{1,i,t}^*, \quad (37)$$

$$d\hat{y}_{i,t} = (-\kappa \hat{y}_{i,t})dt + \sigma_2 dz_{2,i,t}^*, \quad (38)$$

$$d\hat{v}_{i,t} = [a^3\beta - a\hat{v}_{i,t} + a(1-a)\bar{\hat{v}} - a\sigma_{13}]dt + \sigma_3 dz_{3,i,t}^*, \quad (39)$$

With this equations and some initial conditions for S_0 , $\hat{y}_0$, and $\hat{v}_0$, we can generate every i path, with $i \in \{1, \dots, M\}$, at time t for the spot price of the commodity $S_{i,t}$, and therefore construct

the futures curve for each point using equation (40).

$$F_i(t_j, t_l) = S \cdot \exp \left[-\hat{y}_{i,t_j} \frac{1 - e^{-\kappa t_l}}{\kappa} + \hat{v}_{i,t_j} \frac{1 - e^{-at_l}}{a} + B(t_l) \right], \quad (40)$$

for a futures contract at time t_j maturing at t_l , where $B(t_l)$ is defined by,

$$B(t_l) = -\frac{\sigma_{12}}{\kappa^2} (\kappa t_l + e^{-\kappa t_l} - 1) + \frac{\sigma_2^2}{4\kappa^3} (-e^{-2\kappa t_l} - 4e^{-\kappa t_l} + 2\kappa t_l - 3) + \beta (at_l + e^{-at_l} - 1) \\ - \frac{\sigma_3^2}{4a^2} (e^{-2at_l} - 4e^{-at_l} - 2at_l + 3) - \frac{\sigma_{23}}{\kappa^2 a^2 (\kappa + a)} \begin{pmatrix} \kappa^2 e^{-at_l} + \kappa a e^{-at_l} + \kappa a^2 t_l \\ + \kappa^2 at_l + \kappa a e^{-\kappa t_l} + a^2 e^{-\kappa t_l} \\ - \kappa a e^{-(\kappa+a)t_l} - \kappa^2 \\ - \kappa a - a^2 \end{pmatrix} \quad (41)$$

A.3 The parameters of these models

In table 5 we can look at the parameters used in the generation of prices and futures curves. For the oil price model, we used the same parameters used in the paper by Schwartz and Trolle (2010)—that is, the single stochastic volatility model—so that results remain comparable in the baseline case. In the case of the copper price model, the parameters were found using the methodology detailed in the paper by Cortázar and Schwartz (2003), using daily copper prices and their associated futures contracts.³⁰

³⁰Price data was taken from Bloomberg from 3,375 daily observations of copper prices and futures contracts expiring in 1, 2, 3, 6, 9, 12, 36, 48, 60, 72, 84, 96, 108 and 120 months.

Table 5: Parameters used in the path generation process described in A.1 and A.2

Value of the parameters			
Trolle and Schwartz (2009) Crude oil		Cortázar and Schwartz (2003) Copper	
κ	1.0125	κ	2.1354
σ_v	2.8226	a	0.1344
α	0.1365	β	-0.3430
γ	0.7796	σ_1^2	0.0984
σ_S	0.2275	σ_2^2	0.4238
φ	0.0054	σ_3^2	0.0021
ρ_{12}	-0.8797	ρ_{12}	0.9900
ρ_{13}	-0.0912	ρ_{13}	-0.4944
ρ_{23}	-0.1128	ρ_{23}	-0.4887
η	1		
r	0.05	r	0.05
ϕ_0	0.33	$\hat{y}_0$	-0.07
v_0	2.79	$\bar{\hat{v}}^\dagger$	0.025

$^\dagger \bar{\hat{v}}$ is the long term average of variable $\hat{v}$ as calculated from available data.

A.4 Final calibration

As we are assuming a risk-neutral process, the no-arbitrage condition implies that at time t_j the expected spot price of the commodity at time t_l is equivalent to the futures price of the commodity maturing at the same price, so that we have

$$F(t_j, t_l) = E_{t_j}[S_{t_l}] \quad (42)$$

This is implicitly assumed in the construction of the models and in the definition of the futures contract price in equations (30) and (40). However, as the generated paths use a discrete time variation, Δt , the identity (42) does not verify exactly. This was necessary in the original valuation model by Schwartz and Trolle (2010) model so that V_{firm}^{exp} and V_{firm} ,³¹ as well as V_{gov}^{exp} and V_{gov} ³² remain equivalent at time t_0 if there is no risk of expropriation.

Therefore, we introduced a calibration parameter in the data process of creating these prices and futures contracts so that the generated spot prices $S_{i,t}$ are being represented unbiasedly by the futures contracts, and so equation (42) verifies for all maturities, t_l , in future contracts of the form $F(t_0, t_l)$. We thus have a calibrated spot price S_{i,t_n}^* given by the following equation,

$$S_{i,t_n}^* = S_{i,t_n} \cdot F(t_0, t_n) \left(\frac{1}{M} \sum_{j=1}^M S_{j,t_n} \right)^{-1} \quad (43)$$

³¹From equations (17) and (19) at page 14.

³²From equations (18) and (20) at the same page as above.

we then replace the generated spot price with the calibrated price $S_{i,t}^*$ for all purposes in the model.